

ACTA ACADEMIAE STROMSTADIENSIS

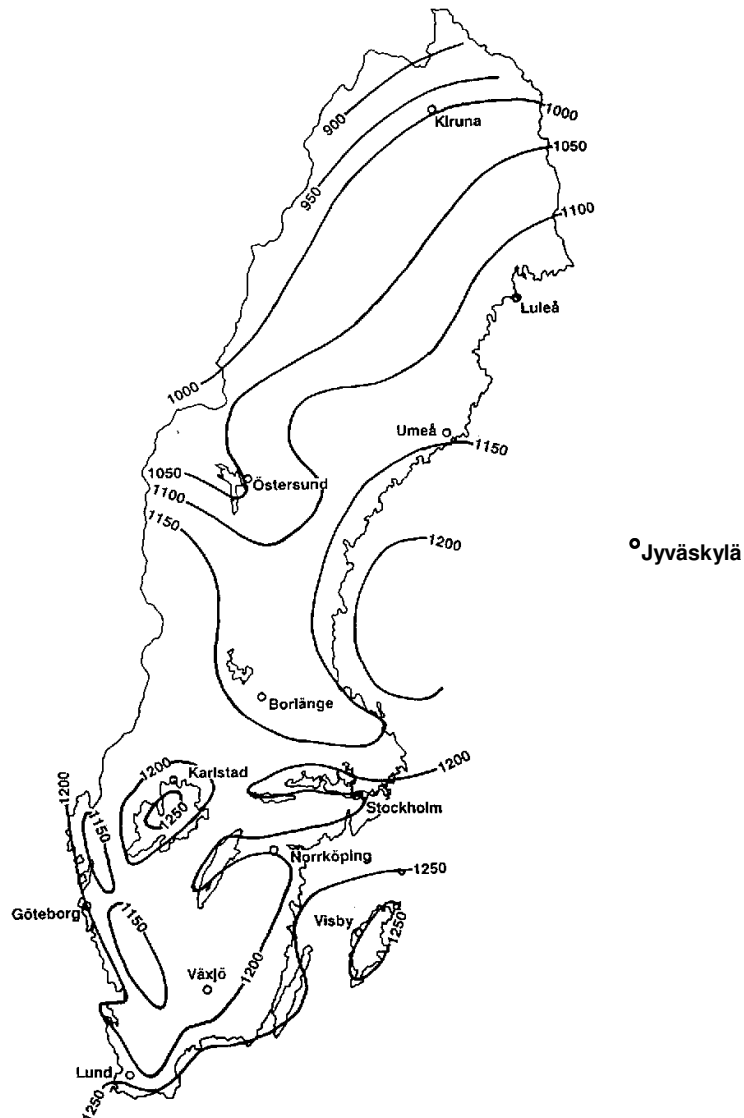
Lars Broman



**Solar Engineering
a Condensed Course**

Lars Broman

Solar Engineering - A Condensed Course



**Solar Thermal Engineering according to Duffie and Beckman,
and Solar Photovoltaic Engineering according to Martin Green**

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Foreword

In 1990, I had invited John Duffie from Solar Laboratory, University of Wisconsin, as Guest Professor at Solar Energy Research Center, Dalarna University, to give a course in thermal solar energy engineering. The course became the final test of a third edition of his and William Beckman standard book to be published in the fall of 1990. Based on their book and my lecture notes, I gave a course several times to engineering students during the coming years and developed it into a 5-week full-time course at MSc and PhD level, first given as a summer course at Tingvall in 1998. Next summer, a similar summer course in photovoltaics based on the books on photovoltaics by Martin Green was given and in the fall the European Solar Engineering School started its 1-year master level program, where one of the courses was thermal solar energy engineering, and another one PV solar energy engineering; both based on the previous experiences.

Then, in 2000, I was asked to give a shorter course, equivalent to 2 weeks full-time study, in solar energy engineering at the Royal Institute of Technology KTH, Stockholm. I built this course on the thermal and PV ESES courses, concentrating on the most important parts but still giving a sound theoretical background. Over the years, I gave this course three more times at KTH and once at a University of Jyväskylä in Finland, gradually developing it into its present form. The present publication has until now been unpublished, used only as a course compendium. I hope that others now may find the text useful.

Lars Broman, Falun 21 November 2011

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(DB) refers to the corresponding sections in Duffie, J. A. and Beckman, W. A., *Solar Engineering of Thermal Processes*, John Wiley & Sons (3rd Ed. 2006).

(AP) refers to the corresponding sections in Wenham, S. R., Green, M. A., and Watt, M. E., *Applied Photovoltaics*, University of New South Wales, Sydney, Australia (1995). Some information is also included from Green, M. A., *Solar Cells* (1992).

All illustrations by L Broman unless otherwise noted. Front page illustration indicates average yearly insolation in kWh/m² on a surface that is tilted 30° towards south.

Chapter 1

Solar Radiation

DB Ch 1-2

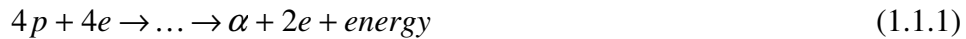
1.1 The Sun

DB 1.1-4

Sun's diameter = 1.4×10^9 m (approx. 100× earth's dia.)

Average distance = 1.5×10^{11} m (approx. 100× sun's dia.)

The sun converts mass into energy (according to Einstein's equation $E = mc^2$) by means of nuclear fusion:



The energy radiates from the sun's surface (the photosphere at approx. 6000 K) mainly as electromagnetic radiation. The sun's power = 3.8×10^{26} W, out of which the earth is irradiated with 1.7×10^{17} W.

The solar constant G_{sc} equals the average power of the sun's radiation that reaches a unit area, perpendicular to the rays, outside the atmosphere (thus *extraterrestrial* or *ET*), at earth's average distance from the sun:

$$G_{sc} = 1367 \text{ [W/m}^2\text{]} (\pm 1 \% \text{ measured uncertainty}) \quad (1.1.2)$$

Note: The letter G is for irradiance = the radiative power per unit area and index sc is for *solar constant*.

The ET solar spectrum is close to the spectrum of a blackbody at 5777 K.

Exercise 1.1.1

Calculate the fraction and the power of the ET solar radiation that is ultraviolet ($\lambda < 0.38 \mu\text{m}$), visible ($0.38 \mu\text{m} < \lambda < 0.78 \mu\text{m}$), and infrared ($\lambda > 0.78 \mu\text{m}$) using the blackbody spectrum tables in Appendix 1.

The sun-earth distance varies $\pm 1.65 \%$ (2.5×10^9 m) - shortest around 1 January - giving a yearly variation of G_{0n} :

$$G_{0n} = 1367 \left(1 + 0.033 \cos \frac{360n}{365} \right) \quad \text{[W/m}^2\text{]} \quad (1.1.3)$$

where n is the day number, index 0 (zero) is for *ET* (no atmosphere), and index n is for *normal* (\perp to the rays).

1.2 Definitions

DB 1.5, 2.1

Air mass AM or m ; $m \approx 1/\cos \theta_z$ where θ_z = the sun's zenith angle.

Beam Radiation = radiation directly from the sun (creates shadows); index b . Radiation on a plane normal to the beam has also index n .

Diffuse Radiation = radiation from the sun whose direction has been changed; also called sky radiation; index d .

Total Solar Radiation = beam + diffuse radiation on a surface; no index. If on a tilted surface, index T .

Global Radiation = total solar radiation on a horizontal surface; no index.

Irradiance or **intensity** of solar radiation G [W/m^2].

Insolation I [$\text{J}/\text{m}^2, \text{hour}$], H [$\text{J}/\text{m}^2, \text{day}$], \bar{H} [$\text{J}/\text{m}^2, \text{day}$; monthly average].
Swedish weather data: H [$\text{Wh}/\text{m}^2, \text{day}$], M [$\text{Wh}/\text{m}^2, \text{month}$].

Solar time = *standard time* corrected for local longitude (+4 min. per degree east and -4 min. per degree west of standard meridian for the local time zone) and time equation E (varies between +15 min. in October and -15 min. in February due to earth's axis tilt and elliptic orbit). During the summer, one more hour has to be subtracted from the daylight saving time. In the following, all times are assumed to be *solar time*.

Solar radiation = short wave radiation, $0.3\mu\text{m} < \lambda < 3\mu\text{m}$

Long wave radiation, $\lambda > 3\mu\text{m}$

Pyrheliometer measures beam (direct) radiation $\{bn\}$ at normal incidence.

Pyranometer measures global $\{b + d\}$ or total $\{bT + dT\}$ radiation.

Exercise 1.2.1.

M for Borlänge in July is $159 \text{ kWh}/\text{m}^2$ (average over many years). What is \bar{H} for that place and month?

1.3 Direction of Beam Radiation

DB 1.6

ϕ = latitude. Latitudes for Swedish solar measurement stations are given in Appendix 2.

δ = the sun's declination (above or below the celestial equator):

$$\delta = 23.45 \sin\left(360 \frac{284 + n}{365}\right) \quad (1.3.1)$$

β = the collector's tilt measured towards the horizontal plane.

γ = the solar collector's azimuth angle = deviation from south, positive towards west, negative towards east.

γ_s = the sun's azimuth angle.

ω = the sun's hour angle measured in degrees ($15^\circ/\text{h}$) from the meridian; positive in the afternoon, negative in the morning.

θ = angle of incidence = angle between the solar collector normal and the (beam) radiation.

θ_z = (the sun's) zenith angle = $90^\circ - \alpha_s$ (solar altitude angle).

n = the day in the year (day number); for monthly average days, dates and declinations, see Appendix 4.

θ is a function of five variables:

$$\begin{aligned} \cos \theta = & \sin \delta \sin \phi \cos \beta - \sin \delta \cos \phi \sin \beta \cos \gamma \\ & + \cos \delta \cos \phi \cos \beta \cos \omega + \cos \delta \sin \phi \sin \beta \cos \gamma \cos \omega \\ & + \cos \delta \sin \beta \sin \gamma \sin \omega \end{aligned} \quad (1.3.2)$$

Exercise 1.3.1

Calculate the angle of incidence of beam radiation on a surface located in Stockholm on 9 November at 1300 (solar time). Surface tilt is 30° towards south-southwest (i. e. 22.5° west of south).

For a collector that is tilted towards south, $\gamma = 0^\circ$, and Equation 1.3.2 is simplified into

$$\begin{aligned} \cos \theta = & \sin \delta \sin \phi \cos \beta - \sin \delta \cos \phi \sin \beta \\ & + \cos \delta \cos \phi \cos \beta \cos \omega + \cos \delta \sin \phi \sin \beta \cos \omega \end{aligned} \quad (1.3.3)$$

For a horizontal surface, $\theta = \theta_z$ and $\beta = 0^\circ$, which inserted into Equation 1.3.3 gives

$$\cos \theta_z = \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega \quad (1.3.4)$$

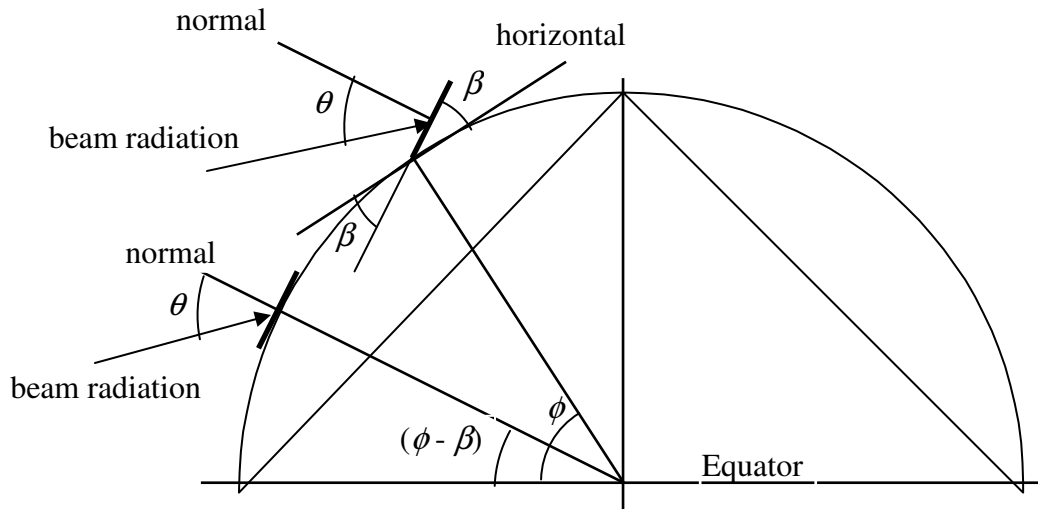


Figure 1.3.1.

South tilted surface with tilt β at latitude ϕ has the same incident angle θ as a horizontal surface at latitude $(\phi - \beta)$:

$$\cos \theta = \cos(\phi - \beta) \cos \delta \cos \omega + \sin(\phi - \beta) \sin \delta \quad (1.3.5)$$

At 12 noon solar time, $\omega = 0$, and

$$\theta_{noon} = |\phi - \beta - \delta| \quad (1.3.6)$$

The hour angle at sunset ω_s is given by Equation 1.3.4 for $\theta_z = 90^\circ$:

$$\cos \omega_s = -\frac{\sin \delta \sin \phi}{\cos \delta \cos \phi} = -\tan \delta \tan \phi \quad (1.3.7)$$

and, similarly, the hour angle ω_s^* for "sunset" for a south tilted surface is given by Equation 1.3.5 for $\theta = 90^\circ$:

$$\cos \omega_s^* = -\tan \delta \tan(\phi - \beta) \quad (1.3.8)$$

unless the sun doesn't set *for real* before then!

Exercise 1.3.2

- (a) at what times (solar time) does the sun set in Stockholm on 20 July and 9 November?
 (b) At what times (solar time) does the sun stop to shine (with beam radiation) onto a surface in Stockholm that is tilted 60° towards south?

From Equation 1.3.7, it is seen that the length of the day N is given by

$$N = \frac{2}{15} \arccos(-\tan \delta \tan \phi) \quad [\text{hours}] \quad (1.3.9)$$

Exercise 1.3.3

Calculate the length of the day in Stockholm on 9 November.

1.4 Ratio of Beam Radiation on Tilted Surface to that on Horizontal Surface, R_b

DB 1.8

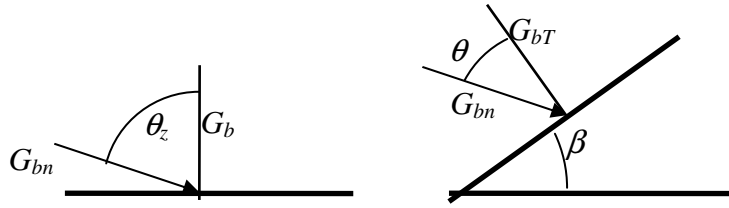


Figure 1.4.1. Beam radiation on horizontal and tilted surfaces.

From Figure 1.4.1, it is seen that the ratio R_b is given by

$$R_b = \frac{G_{bT}}{G_b} = \frac{G_{bn} \cos \theta}{G_{bn} \cos \theta_z} = \frac{\cos \theta}{\cos \theta_z} \quad (1.4.1)$$

For insertion in Equation 1.4.1, $\cos \theta$ and $\cos \theta_z$ are calculated using equations 1.3.2 and 1.3.4, respectively.

Exercise 1.4.1

What is the ratio of beam radiation to that on a horizontal surface for the surface, time, and date given in Exercise 1.3.1?

For a south tilted surface, $\cos \theta$ is given in the simplest way by Equation 1.3.5, giving

$$R_b = \frac{\cos(\phi - \beta) \cos \delta \cos \omega + \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta} \quad (1.4.2)$$

1.5 ET Radiation on Horizontal Surface, G_0

DB 1.10

As will be seen below, the extra-terrestrial solar radiation on a horizontal surface is a useful quantity (see Section 1.8). The power of the radiation is given by

$$G_0 = G_{0n} \cos \theta_z \quad (1.5.1)$$

where G_{0n} is given by Equation 1.1.3 and $\cos \theta_z$ by Equation 1.3.4.

Integration of Equation 1.5.1 from $-\omega_s$ to $+\omega_s$ (see Equation 1.3.7) gives the daily energy H_0 :

$$H_0 = \frac{24 \times 3600 G_{0n}}{\pi} \left(\cos \phi \cos \delta \sin \omega_s + \frac{\pi \omega_s}{180} \sin \phi \sin \delta \right) \quad (1.5.2)$$

\bar{H}_0 is approximately equal to H_0 for the month's average day (see Appendix 4) and M_0 equals \bar{H}_0 multiplied by the number of days in the month (and converted from MJ/day to kWh/mo.)

Exercise 1.5.1

What is H_0 for Stockholm on 14 November?

Exercise 1.5.2

What is M_0 for Stockholm and the month of November?

1.6 Atmospheric Attenuation of Solar Radiation

DB 2.6

Sweden has 13 meteorological stations with insolation data; see Appendix 2 and 3.

Figure 1.6.1 shows how the sun's radiation is attenuated through Rayley scattering and absorption in O_3 , H_2O and CO_2 :

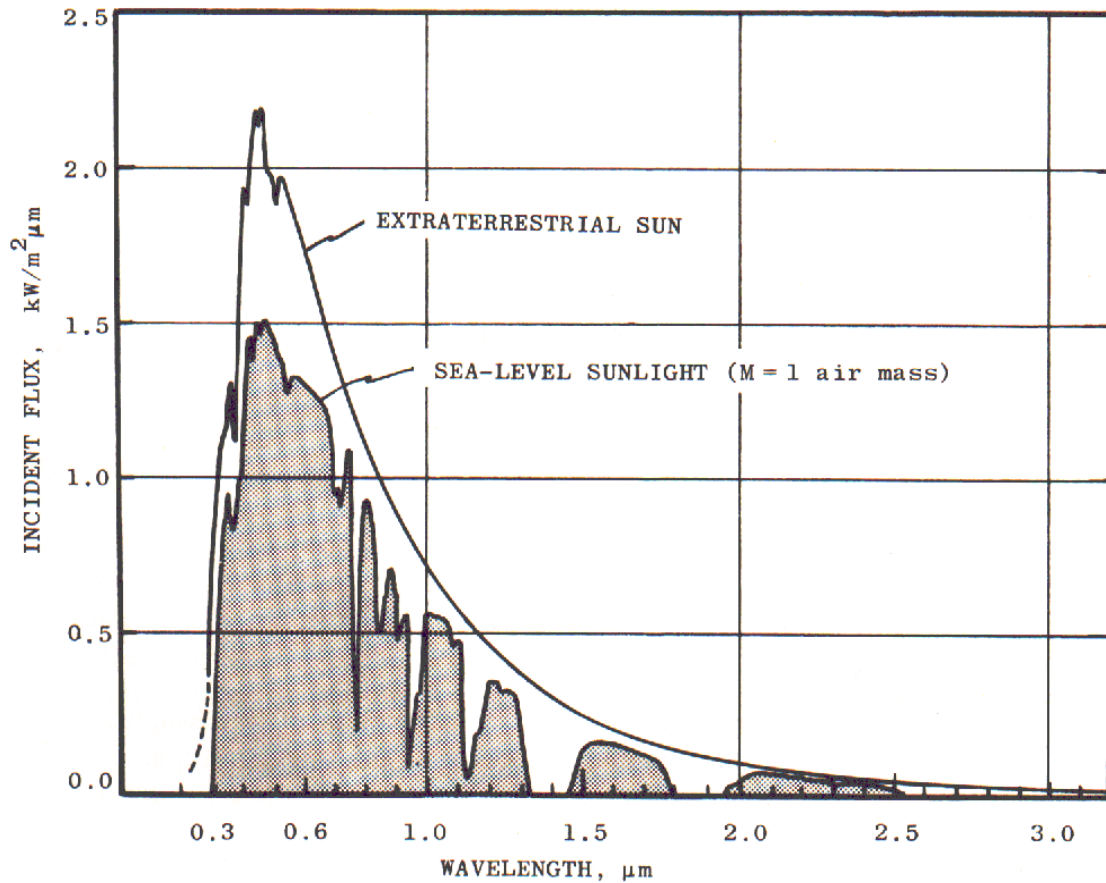


Figure 1.6.1 (from Duffie-Beckman)

This figure is for AM 1. Attenuation is larger for AM 1.5 and AM 2. Since the Rayleigh scattering is higher for lower wavelengths, the diffuse sky radiation has an intensity maximum at $0.4 \mu\text{m}$, making the clear sky blue. The spectral distribution of terrestrial beam radiation at AM 2 (and 23 km visibility) is given in the Table in Appendix 5.

1.7 Estimation of Clear Sky Radiation

(Meinel and Meinel, replaces DB 2.8)

The intensity of beam radiation varies with weather, air quality, altitude over sea level, and the sun's zenith angle θ_z . It is therefore impossible to tell the intensity without measuring it. There exists however a formula that gives an approximate estimate at moderate elevations, clear weather and dry air:

$$G_{bn} \approx G_{0n} \exp[-c(1/\cos\theta_z)^s] \quad (1.7.1)$$

where the empirical constants $c = 0.347$ and $s = 0.678$. The intensity of the diffuse sky radiation is about 10 % of the beam radiation for these circumstances, but may be much higher. (The total intensity is seldom over $1.1G_{bn}$.)

Exercise 1.7.1

Estimate G_{bn} when the sun is 25° over the horizon a clear day. Date 16 August.

1.8 Beam and Diffuse Components of Monthly Radiation

DB 2.12

Appendix 3 gives not only monthly global insolation but also the beam and diffuse components for the Swedish solar measurement sites. Normally, however, only global insolation is known. In order to estimate the components one can compare the measured global insolation M with the ET insolation M_0 , given by Equation 1.5.2 (times the number of days in the month). The ratio between M and M_0 is called monthly clearness index K_{TM} :

$$K_{TM} = M/M_0 \quad (1.8.1)$$

Qualitatively, it is obvious that the diffuse fraction M_d/M decreases when K_{TM} increases. Quantitatively, the following equations approximate the relation:

For $\omega_s \leq 81.4^\circ$ and $0.3 \leq K_{TM} \leq 0.8$

$$\frac{M_d}{M} = 1.391 - 3.560K_{TM} + 4.189K_{TM}^2 - 2.137K_{TM}^3 \quad (1.8.2 \text{ a})$$

For $\omega_s \geq 81.4^\circ$ and $0.3 \leq K_{TM} \leq 0.8$

$$\frac{M_d}{M} = 1.311 - 3.022K_{TM} + 3.427K_{TM}^2 - 1.821K_{TM}^3 \quad (1.8.2 \text{ b})$$

[Ref. Erbs, D. G., Klein, S. A., and Duffie, J. A., *Solar Energy* **28**(1982)293]

Exercise 1.8.1

Using the Erbs *et al.* formula, estimate the diffuse and beam fractions of global radiation in Stockholm for November. How well does the formula estimate the measured fractions?

1.9 Radiation on Sloped Surfaces - Isotropic Sky

DB 2.15,19,21

The sky is brighter near the horizon and around the sun; isotropic sky is however an acceptable approximation. Under this assumption, I_T for a tilted surface is given by

$$I_T = I_b R_b + I_d \left(\frac{1 + \cos \beta}{2} \right) + I \rho_g \left(\frac{1 - \cos \beta}{2} \right) \quad (1.9.1)$$

where $R_b = \cos \theta / \cos \theta_z$ is given by Equation 1.4.2 (for a south tilted surface) and ρ_g is the ground albedo (reflectance).

Define $R = I_T/I$. This gives

$$R = \frac{I_b}{I} R_b + \frac{I_d}{I} \left(\frac{1 + \cos \beta}{2} \right) + \rho_g \left(\frac{1 - \cos \beta}{2} \right) \quad (1.9.2)$$

For monthly insolation on a tilted surface, we similarly get

$$R_M = \frac{M_T}{M} = \frac{M_b}{M} R_{Mb} + \frac{M_d}{M} \left(\frac{1 + \cos \beta}{2} \right) + \rho_g \left(\frac{1 - \cos \beta}{2} \right) \quad (1.9.3)$$

where M_d/M is a function of K_{TM} , Equation 1.8.2 (or calculated from values given in Appendix 3).

For a south tilted surface,

$$R_{Mb} = \frac{2 \int_0^{\omega_s'} \cos \theta d\omega}{2 \int_0^{\omega_s'} \cos \theta_z d\omega} \quad (1.9.4)$$

where $\cos \theta_z$ is given by Equation 1.3.4 and, for a south tilted surface, $\cos \theta$ by Equation 1.3.5; ω_s' is the sun's hour angle for "sunset" for a tilted surface (i. e. smallest angle of ω_s and ω_s^*), giving the following equations:

$$R_{Mb} = \frac{\cos(\phi - \beta) \cos \delta \sin \omega_s' + (\pi/180) \omega_s' \sin(\phi - \beta) \sin \delta}{\cos \phi \cos \delta \sin \omega_s + (\pi/180) \omega_s \sin \phi \sin \delta} \quad (1.9.5 a)$$

where

$$\omega_s' = \min[\omega_s = \arccos(-\tan \phi \tan \delta); \omega_s^* = \arccos(-\tan(\phi - \beta) \tan \delta)] \quad (1.9.5 b)$$

Exercise 1.9.1

Estimate the average monthly average radiation incident on a Stockholm collector that is tilted 60° towards south, for the months June and November. Ground reflectance $\rho_g = 0.50$.

Note: For a surface that is tilted to an arbitrary direction, the calculations are a bit more complicated, and usually a simulation program (like TRNSYS) is utilized. However, for a surface at mid-northern latitude and tilted 30 - 60° degrees between SE and SW, the insolation is not less than 95 % of that onto a south tilted surface (for some months and some tilts it can even be higher). See also Appendix 7!

Chapter 2

Selected Heat Transfer Topics

DB Ch 3

Radiation is approximately as important as conduction and convection in solar collectors where the energy flow per m^2 is about two orders of magnitude lower than for "conventional" processes (flat plate collector max. $1 \text{ kW}/m^2$, electric oven hob typically $1 \text{ kW}/dm^2$).

2.1 Electromagnetic Radiation

DB 3.1-6

The electromagnetic spectrum

Emission of thermal radiation is due to electrons, atoms and molecules changing energy state in a heated material; the emission is typically over a broad energy interval. The radiation is characterized by wavelength λ [m], frequency ν [Hz], and speed $c_1 = c/n$ [m/s] where c = the speed of light in vacuum and n = the refractive index of the material:

$$\lambda \cdot \nu = c_1 = c/n \quad (2.1.1)$$

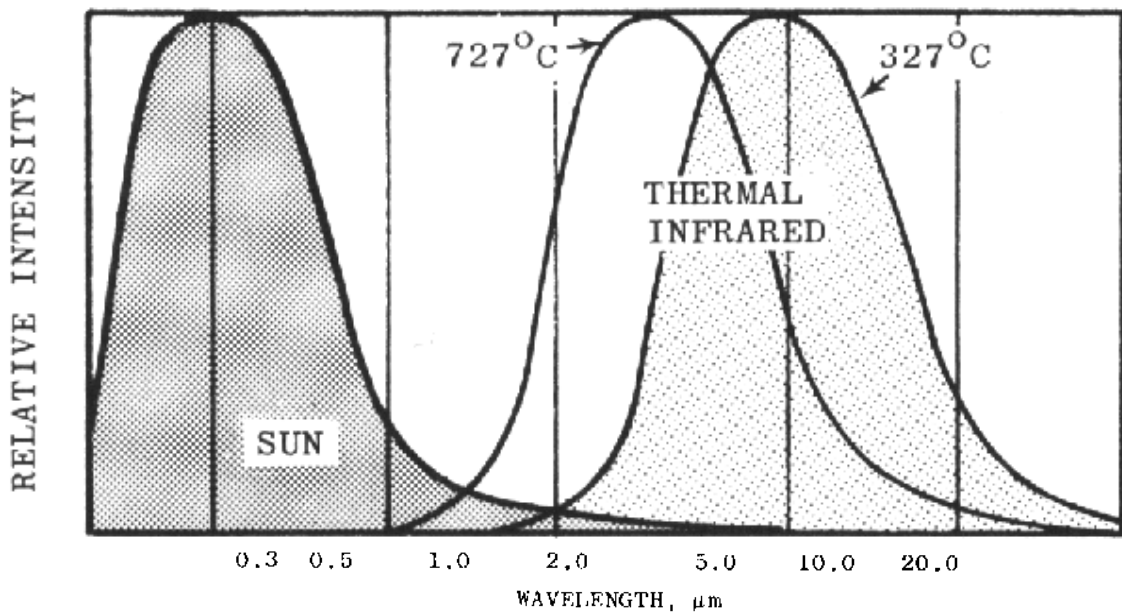


Figure 2.1.1 Three electromagnetic spectra. Visible light is the interval $0.38 < \lambda < 0.78 \mu\text{m}$, ultra violet light $\lambda < 0.38 \mu\text{m}$, and infrared radiation $\lambda > 0.78 \mu\text{m}$ (from Duffie-Beckman).

Photons

Light consists of photons, whose energy E is related to the frequency ν :

$$E = h \cdot \nu \quad (2.1.2)$$

where Planck's constant $h = 6.6256 \cdot 10^{-34}$ [Js]

Blackbody radiation

An ideal blackbody absorbs *and* emits the maximum amount of radiation: Cavity 100 %, "pitch black" 99 %, "black" paint 90-95 %.

Planck's radiation law

Thermal radiation has wavelengths between 0.2 μm (200 nm) and 1000 μm (1 mm). The spectrum of blackbody radiation is, according to Planck:

$$\frac{dE}{\partial A \partial \lambda} = E_{\lambda b} = \frac{C_1}{\lambda^5 [\exp(C_2 / \lambda T) - 1]} \quad (2.1.3)$$

where $C_1 = 3.74 \cdot 10^{-16}$ [m^2W] and $C_2 = 0.0144$ [$\text{m}\cdot\text{K}$].

Wien's displacement law

Derivation of Planck's radiation law gives Wien's displacement law:

$$\lambda_{max} \cdot T = 2898 \text{ } [\mu\text{m}\cdot\text{K}]. \quad (2.1.4)$$

Stefan-Boltzmann's radiation law

Integration of Planck's radiation law gives Stefan-Boltzmann's radiation law:

$$\frac{dE}{dA} = E_b = \sigma T^4 \quad (2.1.5)$$

where Stefan-Boltzmann's constant $\sigma = 5.67 \cdot 10^{-8}$ [$\text{W}/\text{m}^2, \text{K}^4$].

Radiation tables

The blackbody spectrum is tabled in Appendix 1.

Table 1 gives the fraction of blackbody radiant energy Δf between previous λT and present λT [$\mu\text{m K}$] for different λT -values.

Table 2 gives the fraction of blackbody radiant energy Δf between zero and λT [$\mu\text{m K}$] for even fractional increments.

Exercise 2.1.1

Assume the sun is a blackbody at 5777 K. (a) What is the wavelength at which the maximum monochromatic emissive power occurs? (b) At what wavelength λ_m is half of the emitted radiation below λ_m and half above λ_m (λ_m = "median wavelength").

2.2 Radiation Intensity and Flux

DB 3.7

In this Section, intensity and flux are defined in a general sense, i. e. for radiation emitted, absorbed or just passing a real or imaginary plane.

$$\text{Intensity } I: \quad I = \frac{dE}{\partial A \partial \omega} \perp \text{ mot } A \quad [\text{W/m}^2, \text{sterradian}] \quad (2.2.1)$$

$$\text{Flux } q: \quad q = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I \cos \theta \sin \theta d\theta d\phi \quad [\text{W/m}^2] \quad (2.2.2)$$

Here, θ is the exit (or incident) angle (measured from the normal) and ϕ is the "azimuth" angle. For the special case a surface where $I = \text{constant}$ independently of θ and ϕ , integration gives:

$$q = \pi \cdot I \quad (2.2.3)$$

Such a surface (where $I = \text{constant}$) is called diffuse or Lambertian. An ideal blackbody is diffuse:

$$I_b = E_b / \pi \quad (2.2.4)$$

This applies also to monochromatic radiation, so for a particular λ we get:

$$I_{\lambda b} = E_{\lambda b} / \pi \quad (2.2.5)$$

Equation 2.2.4 complements 2.1.5 (Stefan-Boltzmann), and Equation 2.2.5 complements 2.1.3 (Planck).

Integrating (2.2.2) over all ϕ gives $\frac{dq}{d\theta} = \pi \cdot I \cdot \sin 2\theta$ (for a Lambertian surface).

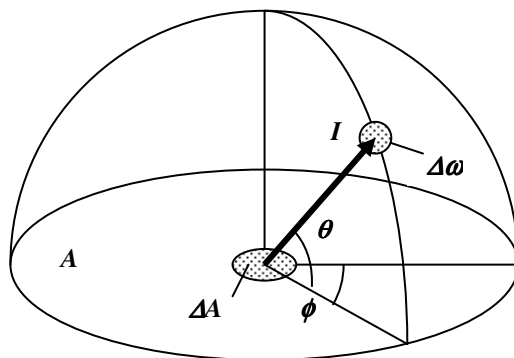


Figure 2.2.1

2.3 IR Radiation Exchange Between Gray Surfaces

DB3.8

We identify two special (idealized) cases.

(1) Exchange of radiation between two large parallel surfaces:

$$\frac{Q}{A} = \frac{\sigma(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \quad [\text{W/m}^2] \quad (2.3.1)$$

(2) Exchange of radiation between a small object (A_1) and a large enclosure:

$$Q_1 = \varepsilon_1 A_1 \sigma(T_1^4 - T_2^4) \quad [\text{W}] \quad (2.3.2)$$

2.4 Sky Radiation

DB 3.9

A solar collector exchanges radiative energy with the surroundings according to Equation 2.3.2:

$$Q = \varepsilon A \sigma (T^4 - T_s^4) \quad (2.4.1)$$

Here, $T_s = T_a \cdot C_{corr.}$ (2.4.2)

where $C_{corr.}$ varies with the humidity of the of the air (≈ 1 when humid or cloudy, ≈ 0.9 when clear and dry air). Usually, $C_{corr.} = 1$ is a good enough approximation.

2.5 Radiation Heat Transfer Coefficient

DB 3.10

We want Equation 2.3.1 written as

$$Q = A_1 h_r (T_2 - T_1) \quad (2.5.1)$$

which, from Equation (2.3.1), gives the heat transfer coefficient h_r :

$$h_r = \frac{\sigma(T_2^2 + T_1^2)(T_2 + T_1)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} \quad (2.5.2)$$

In Equation (2.5.2), the nominator can be approximated by $4\sigma\bar{T}^3$ where \bar{T} is the average of T_2 and T_1 . (Verify this!)

Exercise 2.5.1

The plate and cover of a flat-plate collector are large in extent, parallel, and 25 mm apart. The emittance of the plate is 0.15 and its temperature is 70°C. The emittance of the glass cover is 0.88 and its temperature 50°C. Calculate the radiation exchange between the surfaces Q/A and the heat transfer coefficient h_r .

2.6 Natural Convection Between Flat Parallel Plates

DB 3.11-12

The dimensionless Raleigh number Ra is a function of the gas' (usually the air's) properties (at the actual temperature) and the temperature difference between the plates ΔT :

$$Ra = \frac{g \cdot \Delta T \cdot L^3}{\nu \cdot \alpha \cdot \bar{T}} \quad (2.6.1)$$

where g = gravitational constant (9.81 m/s²), L = plate spacing, α = thermal diffusivity, and ν = kinematic viscosity (= $Pr \cdot \alpha$ where Pr is the dimensionless Prandtl number).

For parallel plates, the dimensionless Nusselt number $Nu = 1$ for pure conduction, and when both conduction and convection takes place, given by

$$Nu = \frac{h}{k/L} = \frac{hL}{k} \quad (2.6.2)$$

where h = heat transfer coefficient and k = thermal conductivity.

Some useful properties of air are found in Appendix 6.

When convection takes place, Nu is given by Equation 2.6.3:

$$Nu = 1 + 1.44 \left[1 - \frac{1708(\sin 1.8\beta)^{1.6}}{Ra \cos \beta} \right] \left[1 - \frac{1708}{Ra \cos \beta} \right]^+ + \left[\left(\frac{Ra \cos \beta}{5830} \right)^{1/3} - 1 \right]^+ \quad (2.6.3)$$

where the meaning of the + exponent is that only positive values of the square brackets are to be used (i. e., use zero if bracket is negative).

Exercise 2.6.1

Find the convection heat transfer coefficient h (including conduction!) between two parallel plates separated by 25 mm with 45° tilt. The lower plate is at 70°C and the upper plate at 50°C.

Note: The curve for 75° tilt of the solar collector is also good for vertical (tilt 90°).

Convection in a solar collector can be suppressed by various means like honeycomb and aerogel. Most common is a flat film between the glazing and the absorber plate.

Exercise 2.6.2

What would h in Exercise 2.6.1 approximately become if a flat film is added between the glazing and the plate? Assume that the temperature of the film is 60°C.

2.7 Wind Convection Coefficients

DB 3.15

Recommendation:

$$h_w = \max \left[5, \frac{8.6V^{0.6}}{L^{0.4}} \right] \quad [\text{W/m}^2\text{C}] \quad (2.8.1)$$

where h_w = the heat transfer coefficient for wind.

for wind speed V [m/s] and a collector on a house with $L = \sqrt[3]{\text{volume}}$.

World average value $V = 5$ m/s $\Rightarrow h_w \approx 10$ [W/m²C] (see also Exercise 2.7.1).

When only absorber and ambient temperatures are known, but not the glazing's temperature T_g (and this is of course normally the case!) the procedure is to *guess* T_g , then calculate the radiative and convective losses both absorber \rightarrow glazing and glazing \rightarrow ambient, and keep adjusting T_g until the two match. For more information see Section 5.3 (and Duffie-Beckman, Chapter 6).

Exercise 2.7.1

- (a) What L makes $h_w = 10$ W/m²,K for $V = 5$ m/s?
- (b) How much will h_w change if L is doubled or halved?

Chapter 3

Radiation Characteristics for Opaque Materials

DB Ch 4

3.1 Absorptance, Emittance and Reflectance

DB 4.1-6, 11, 13

Absorptance and emittance

$\varepsilon_\lambda / \alpha_\lambda(\mu, \phi)$ = emittance/absorptance at wavelength λ and direction θ, ϕ ($\mu = \cos \theta$).

$\varepsilon / \alpha(\mu, \phi)$ = emittance/absorptance at all wavelengths and direction θ, ϕ .

ε / α = emittance/absorptance at all wavelengths, all directions.

$\varepsilon_\lambda / \alpha_\lambda$ = emittance/absorptance at wavelength λ , all directions.

Absorptance $\alpha_\lambda(\mu, \phi)$ and emittance $\varepsilon_\lambda(\mu, \phi)$ are surface properties:

$$\alpha_\lambda(\mu, \phi) = \frac{I_{\lambda,a}(\mu, \phi)}{I_{\lambda,i}(\mu, \phi)} \quad (3.1.1)$$

$$\varepsilon_\lambda(\mu, \phi) = \frac{I_\lambda(\mu, \phi)}{I_{\lambda b}} \quad (3.1.2)$$

Absorptance α and emittance ε can now be calculated by means of integrating the I :s. The resulting complicated equations are much simplified if we (i) assume that α and ε are independent of μ and ϕ (which is rather true) and (ii) that, in the case of α , we restrict ourselves to the solar spectrum (indicated by subscript s):

$$\alpha = \frac{\int_0^\infty \alpha_\lambda E_{\lambda s} d\lambda}{E_s} \quad (3.1.3)$$

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda b} d\lambda}{E_b} \quad (3.1.4)$$

Note that α and ε are not only dependent on the properties of the surface but also of the spectrum; in the case of ε therefore on the temperature of the radiating surface.

Kirchhoff's law

For a body in thermal equilibrium with a surrounding (evacuated) enclosure, absorbed and emitted energy must be equal. From this fact can we conclude that, in this case, α and ε for this body must be the same. Then this must be true for all λ :s. This is important, because, since α_λ and ε_λ are surface properties only,

$$\varepsilon_\lambda = \alpha_\lambda \quad (3.1.5)$$

must hold for all surfaces.

Reflectance

Reflectance ρ may be specular (as at an ideal mirror), diffuse, or a mixture of both. The monochromatic reflectance ρ_λ is a surface property, but the total reflectance ρ is also dependent on the spectrum.

Relationships between absorptance, emittance, and reflectance

All incident light that is not absorbed is reflected. Therefore,

$$\rho + \alpha = 1 \quad (3.1.6)$$

Restricting ourselves to the monochromatic quantities, emittance can be included:

$$\rho_\lambda + \alpha_\lambda = \rho_\lambda + \varepsilon_\lambda = 1 \quad (3.1.7)$$

Finally, note that while ε is determined by the surface's properties and temperature, α (and thus ρ) depends on an external factor, the spectral distribution of the incident radiation.

Calculation of emittance and absorptance

This is done by integration, usually by means of numerical integration; i. e. summation over a number of equal energy intervals of the spectrum:

$$\varepsilon = \frac{1}{n} \sum_{j=1}^n \varepsilon_{\lambda_j} = 1 - \frac{1}{n} \sum_{j=1}^n \rho_{\lambda_j} \quad (3.1.8)$$

$$\alpha = \frac{1}{n} \sum_{j=1}^n \alpha_{\lambda_j} = 1 - \frac{1}{n} \sum_{j=1}^n \rho_{\lambda_j} \quad (3.1.9)$$

Exercise 3.1.1

Calculate the absorptance α for the terrestrial solar spectrum in Appendix 5 of a (hypothetical) surface with a non-constant $\rho_\lambda = 0.05\lambda$ [λ in μm] by means of numerical integration.

Exercise 3.1.2

Calculate the emittance ε for the same surface at temperature 400 K using Table 2 in Appendix 1.

Angular dependence of solar absorptance

For a typical surface, α decreases at large incidence angles. This will be taken into account in the angular dependence of the transmittance-absorptance product ($\tau\alpha$); see Section 5.6. Also specularly reflecting surfaces may show such decrease, especially degraded surfaces.

3.2 Selective Surfaces

DB 4.8-10

An absorber with large α_λ for the region of the solar spectrum and a small ε_λ for the long-wave region would be very effective. An absorber with such a surface is called selective.

There are some different mechanisms employed in making selective surfaces:

- (a) Thin (a few μm) black surface (black chrome, black nickel, copper oxide, ...) on a reflective surface.
- (b) Enhanced absorptance through successive (specular) reflection in V-troughs.
- (c) Thin black surface + micro structure (like the black nickel sputtered aluminum surface of the Swedish Sunstrip[®] absorber).

Exercise 3.2.1

Calculate the absorptance for blackbody radiation from a source at 5777 K and the emittance at surface temperatures 100 C and 500 C (using Appendix 1, Table 1) for a surface with $\rho_\lambda = 0.1$ for $\lambda < 3 \mu\text{m}$, and $\rho_\lambda = 0.9$ for $\lambda > 3 \mu\text{m}$. (This is a hypothetical, but not terribly unrealistic good selective surface.) Is the emittance dependent on the surface temperature?

Note that a large α (for solar radiation) is even more important than a small ε for thermal radiation. The quantity α/ε is sometimes referred to as the selectivity of the absorber.

Chapter 4

Radiation Transmission Through Glazing; Absorbed Radiation

DB Ch 5

4.1 Reflection of Radiation

DB 5.1

The reflectance r is given by Fresnel's expressions for reflection of unpolarized light passing from one medium with refractive index n_1 to another medium with refractive index n_2 :

$$r_{\perp} = \frac{\sin^2(\theta_2 - \theta_1)}{\sin^2(\theta_2 + \theta_1)} \quad (4.1.1)$$

$$r_{\parallel} = \frac{\tan^2(\theta_2 - \theta_1)}{\tan^2(\theta_2 + \theta_1)} \quad (4.1.2)$$

$$r = \frac{I_r}{I_i} = \frac{1}{2}(r_{\perp} + r_{\parallel}) \quad (4.1.3)$$

where

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad (\text{Snell's law}) \quad (4.1.4)$$

At normal incidence ($\theta = 0$):

$$r(0) = \frac{I_r}{I_i} = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 \quad (4.1.5)$$

$n_1 = 1$ (air) and $n_2 = n$:

$$r(0) = \frac{I_r}{I_i} = \left(\frac{n - 1}{n + 1} \right)^2 \quad (4.1.6)$$

Exercise 4.1.1

Calculate the reflectance of one surface of glass at normal incidence and at $\theta = 60^\circ$. The average index of refraction of glass for the solar spectrum is 1.526.

A slab or film of transparent material has two surfaces:

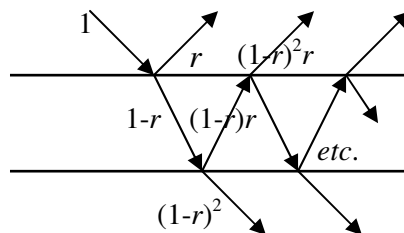


Figure 4.1.1. Transmission through one nonabsorbing cover.

From Figure 4.1.1 we get the following expression for transmission τ :

$$\tau_{\perp} = (1 - r_{\perp})^2 \sum_{n=0}^{\infty} (r_{\perp}^{2n}) = (1 - r_{\perp})^2 / (1 - r_{\perp}^2) = (1 - r_{\perp}) / (1 + r_{\perp}) \quad (4.1.7)$$

Similarly,

$$\tau_{\parallel} = (1 - r_{\parallel}) / (1 + r_{\parallel}) \quad (4.1.8)$$

Note that $r_{\perp} \neq r_{\parallel}$ except for $\theta = 0^\circ$. Finally, for unpolarized light,

$$\tau_r = \frac{1}{2} (\tau_{\perp} + \tau_{\parallel}) \quad (4.1.9)$$

Subscript r shows that this is the transmittance due to reflection.

Exercise 4.1.2

Calculate the transmittance of two covers of nonabsorbing glass at normal incidence and at $\theta = 60^\circ$.

4.2 Optical Properties of Cover Systems

DB 5.2-6

Absorption by glazing is given by

$$dI = -IKdx \quad (4.2.1)$$

where K = extinction coefficient; K is between 4 m^{-1} (for clear white glass) and 32 m^{-1} (for greenish window glass). Integration from 0 to $L/\cos\theta_2$ (where L is the thickness of the cover) gives transmittance due to absorption,

$$\tau_a = \frac{I_{\text{transmitted}}}{I_{\text{incident}}} = \exp\left(-\frac{KL}{\cos\theta_2}\right) \quad (4.2.2)$$

where θ_2 is given by Snell's law (providing θ_1 is known).

With very good approximation, τ for a slab is given by

$$\tau = \tau_a \tau_r \quad (4.2.3)$$

Absorptance and reflectance of a slab is then given by

$$\alpha = 1 - \tau_a \quad (4.2.4)$$

and

$$\rho = 1 - \tau - \alpha = \tau_a - \tau \quad (4.2.5)$$

Exercise 4.2.1

Calculate the transmittance, reflectance, and absorptance of a single glass cover, 3 mm thick, at an angle of 45° . The extinction coefficient of the glass is 32 m^{-1} .

Transmittance of diffuse radiation

Diffuse radiation hits the surface at all incident angles between 0° and 90° . An astonishingly good approximation is to use the effective incident angle $\theta_e = 60^\circ$.

Transmittance-absorptance product ($\tau\alpha$)

Regard ($\tau\alpha$) as *one* symbol for *one* property of the combination glazing + absorber. ($\tau\alpha$) is slightly larger than $\tau\alpha$. A good approximation is

$$(\tau\alpha) = 1.01 \cdot \tau\alpha \quad (4.2.6)$$

Exercise 4.2.2

For a collector with the cover in Exercise 4.2.1 and an absorber plate with $\alpha = 0.90$ (independent of direction), calculate ($\tau\alpha$) for $\theta = 45^\circ$.

The angular dependence of ($\tau\alpha$) is given by the so-called incidence angle modifier; see Section 5.6.

4.3 Absorbed Solar Radiation

DB 5.9

Hourly values of the intensity I_T on a tilted surface are given by Equation 1.9.1 with $R_b = I_{bT} / I_b$ and isotropic diffuse light. Multiplication with appropriate $(\tau\alpha)$ -values yields absorbed radiation S :

$$S = I_b R_b (\tau\alpha)_b + I_d \left(\frac{1 + \cos \beta}{2} \right) (\tau\alpha)_d + \rho_g (I_b + I_d) \left(\frac{1 - \cos \beta}{2} \right) (\tau\alpha)_g \quad (4.3.1)$$

Note that I_b and I_d are for a horizontal surface. In order to calculate $(\tau\alpha)_b$, θ must be known (is also needed to calculate R_b). For $(\tau\alpha)_d$ and $(\tau\alpha)_g$, use $\theta_e = 60^\circ$.

Exercise 4.3.1

For the hour between 11 and 12 on a clear winter day in southern Europe, $I = 1.79 \text{ MJ/m}^2$, $I_b = 1.38 \text{ MJ/m}^2$, and $I_d = 0.41 \text{ MJ/m}^2$. Ground reflectance is 0.6. For this hour, θ for the beam radiation is 17° and $R_b = 2.11$. A collector with one glass cover is sloped 60° to the south. The glass has $KL = 0.0370$ and the absorptance of the plate is 0.93 (at all angles). Using the isotropic diffuse model (Equation 4.3.1), calculate the absorbed radiation per unit area of the absorber.

Equation 4.3.1 is terribly similar to expression 1.9.1 for I_T . It is therefore natural to introduce a new quantity, $(\tau\alpha)_{av}$, defined by

$$S = (\tau\alpha)_{av} I_T \quad (4.3.2)$$

or, for instantaneous values,

$$S = (\tau\alpha)_{av} G_T \quad (4.3.3)$$

4.4 Monthly Average Absorbed Radiation

DB 5.10

The expression for S_M looks like this:

$$S_M = M_b R_{Mb} (\bar{\tau}\alpha)_b + M_d (\bar{\tau}\alpha)_d \left(\frac{1 + \cos \beta}{2} \right) + M \rho_g (\bar{\tau}\alpha)_g \left(\frac{1 - \cos \beta}{2} \right) \quad (4.4.1)$$

Calculation of S_M (for south-tilted surface):

- (1) M is measured (or given, e. g. in Appendix 3).
- (2) M_0 is calculated using Equation 1.5.2 (and following).
- (3) Calculate clearness index $K_{TM} = M / M_0$.
- (4) Finding $M_d / M = f(K_{TM})$ as outlined in Section 1.8. If M_d is known (e. g. from Appendix 3), points (2) - (4) can be omitted.
- (5) $M_b = M - M_d$
- (6) Average incident angle for beam radiation equals approximately θ at 2.5 hours before or after noon on the average day; θ is calculated with Equation 1.3.3.
- (7) θ_e for diffuse and sky components approximately equals 60° .
- (8) τ is calculated for θ and θ_e using the methods in 4.1 and 4.2.
- (9) knowing α , $(\tau\alpha)$ is calculated for the two angles using Equation 4.2.6.
- (10) R_{Mb} is calculated using Equations 1.9.5 a and b.
- (11) Now S_M can be calculated with Equation 4.4.1.

Exercise 4.4.1

Calculate S_M for a 45° south-tilted collector in Stockholm in August. The collector has one glass with $KL = 0.0125$ and α for the absorber is 0.90. Ground reflectance $\rho_g = 0.50$.

Chapter 5 Flat-Plate Collectors

DB Ch 6, 8, 10

5.1 Basic Flat-Plate Energy Balance Equation

DB 6.1-2

A solar collector is (in principle) a heat exchanger radiation \rightarrow (e. g.) hot water. A solar collector is characterized by low and variable energy flow, and that radiation is an important part of the heat balance.

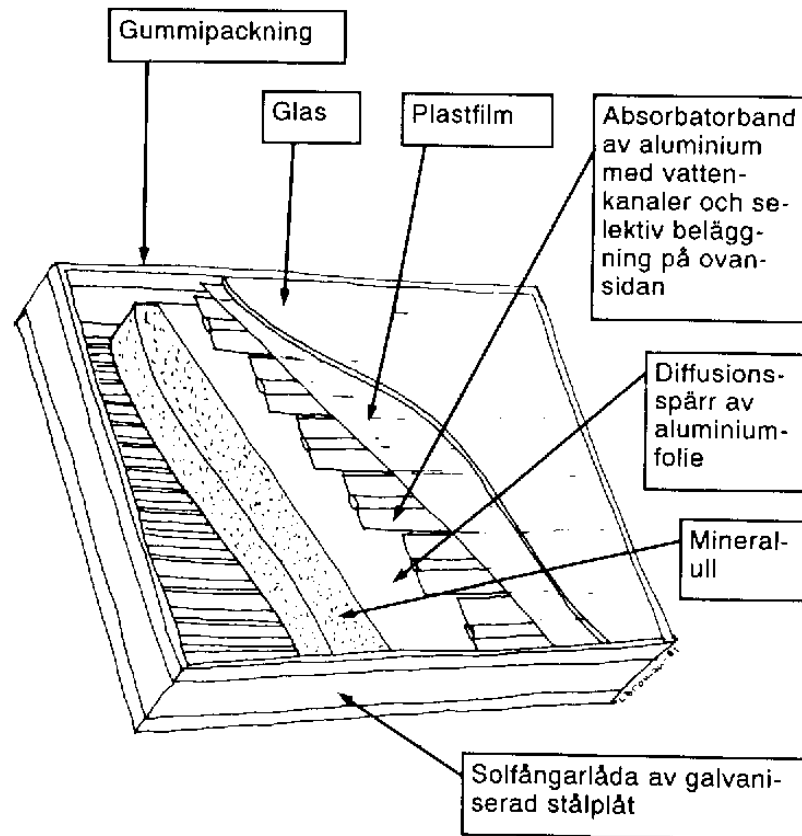


Figure 5.1.1. Typical solar collector. Gummipackning = rubber gasket. Glas = glass. Plastfilm = plastic film. Absorbatorband ... = absorber strips of aluminum with water channels and selective coating on the top side. Diffusionsspärr ... = diffusion barrier made of aluminum foil. Mineralull = mineral wool. Solfångarlåda ... = Collector box of galvanized steel tin.

The useful energy Q_u from a solar collector is given by

$$Q_u = A_c [S - U_L (T_{pm} - T_a)] \quad (5.1.1)$$

where A_c = collector area, S = absorbed energy per m^2 absorber area, U_L = heat loss coefficient, T_{pm} = the absorber surface's average temperature, and T_a = ambient temperature (subscripts p for plate and m for mean value).

Problem: It is difficult both to measure and to calculate T_{pm} . Instead of Equation 5.1.1 we therefore instead use the following expression:

$$Q_u = A_c F_R [S - U_L (T_i - T_a)] \quad (5.1.2)$$

where F_R = the collector's heat removal factor and T_i = the inlet water temperature. Insertion of S from Equation 4.3.3 changes this equation into

$$Q_u = A_c [F_R (\tau\alpha) I_T - F_R U_L (T_i - T_a)] \quad (5.1.3)$$

This is Duffie-Beckman's most important formula. $(\tau\alpha)$ is short for $(\tau\alpha)_{av}$ as defined by Equation 4.3.2/4.3.3.

Normally hourly values are used: S [$\text{J}/\text{m}^2, \text{h}$]; then S is calculated from I_T . Remember that $1 \text{ kWh} = 3.6 \text{ MJ}$.

Especially in tests, instantaneous values are used: S [W/m^2]; then S is calculated from G_T .

The efficiency η of a solar collector is given by

$$\eta = \frac{\int Q_u dt}{A_c \int G_T dt} \quad (5.1.4)$$

5.2 Temperature Distributions in Flat-Plate Collectors

DB 6.3

The temperature T_p of an absorber plate is not constant over the surface, but varies both in parallel with and perpendicular to the water (or fluid) channels.

Under operation, the outlet temperature is higher than the inlet temperature, so absorber temperature increases in parallel with the water flow. Heat is conducted through the absorber towards the water channels, so the absorber temperature perpendicular to the channels is lowest at the channels and highest in the middle between them.

Finally, there is a temperature difference between the channel (tube) and the water. The absorber temperature is therefore on the average higher than the water inlet temperature; hence the factor F_R in Equation 5.1.2.

5.3 Collector Overall Heat Loss Coefficient

DB 6.4

The heat loss coefficient U_L [W/m²K] is the sum of the top, bottom, and edge loss coefficients:

$$U_L = U_t + U_b + U_e \quad (5.3.1)$$

The top loss coefficient is due to convection and radiation, and the two others to heat conduction. For an efficient collector, all three are kept low (as low as it is economical). Bottom and edge losses are minimized by means of adequate insulation.

The top loss coefficient is more difficult to make low without decreasing S . An extra glass or plastic film between the glass and the absorber decreases convection losses, but also S gets lower. A selective absorber surface gives much lower radiation losses than an absorber covered with black paint.

(One more way to minimize losses is to keep the average temperature of the absorber as low as possible, since the heat losses from an absorber are proportional to the difference between this temperature and the ambient temperature.)

U_L can be calculated from the optical, geometrical and thermal properties of the collector, and is typically between 2 and 8 W/m²K. U_L (or, rather, $F_R U_L$) can also be measured, and this is always done by collector manufacturers. In the present treatment it will be assumed that measured heat loss factors are available. The interested reader, who wants to learn the intricacies of calculating a collector's U_L , is referred to Duffie-Beckman.

5.4 Collector Heat Removal Factor F_R

DB 6.5-7

The heat removal factor F_R is the product of two factors, the collector efficiency factor F' and the collector flow factor F'' . F' compensates for the fact that the temperature of an absorber cross section perpendicular to the water flow is higher than the temperature of the water. $F' \approx F_{av}$ (Section 5.6).

F'' compensates for the fact that the average temperature along the water flow T_{av} is higher than the inlet temperature T_i .

5.5 Collector Characterization

DB 6.15-16

Measured collector performance and performance calculated according to the principles mentioned above agree very well. The collector is also well described by the (stationary) Equation 5.1.3, possibly complemented by the collector's dynamic performance.

This equation contains three parameters, U_L and F_R which are constant or varies with temperature, and $(\tau\alpha)$ that is constant or varies with incidence angle θ_i .

The collector's instantaneous efficiency η_i is given by

$$\eta_i = \frac{Q_u}{A_c G_T} = F_R (\tau\alpha) - \frac{F_R U_L (T_i - T_a)}{G_T} \quad (5.5.1)$$

In the next section, the incidence angle modifier

$$K_{\tau\alpha} = (\tau\alpha)/(\tau\alpha)_n = f(\theta_b) \quad (5.5.2)$$

where the parameter b_o is part of the expression, will be explained. This leaves us with three basic solar collector parameters:

- * $F_R(\tau\alpha)_n$ indicating how energy is absorbed;
- * $F_R U_L$ indicating how energy is lost; and
- * b_o indicating the dependence of the incidence angle θ_b .

5.6 Collector Tests

DB 6.17-20

A (complete) solar collector test consists of three measurements:

- * instantaneous efficiency at near normal incidence;
- * dependence on the incidence angle; and
- * the collector's time constant.

In order to determine η_i , mass flow \dot{m} , outlet temperature T_o , and inlet temperature T_i , solar radiation intensity G_T , and wind speed V are measured, giving

$$\eta_i = \frac{\dot{m}C_p(T_o - T_i)}{A_c G_T} \quad (5.6.1)$$

The so obtained η_i is inserted into Equation 5.5.1.

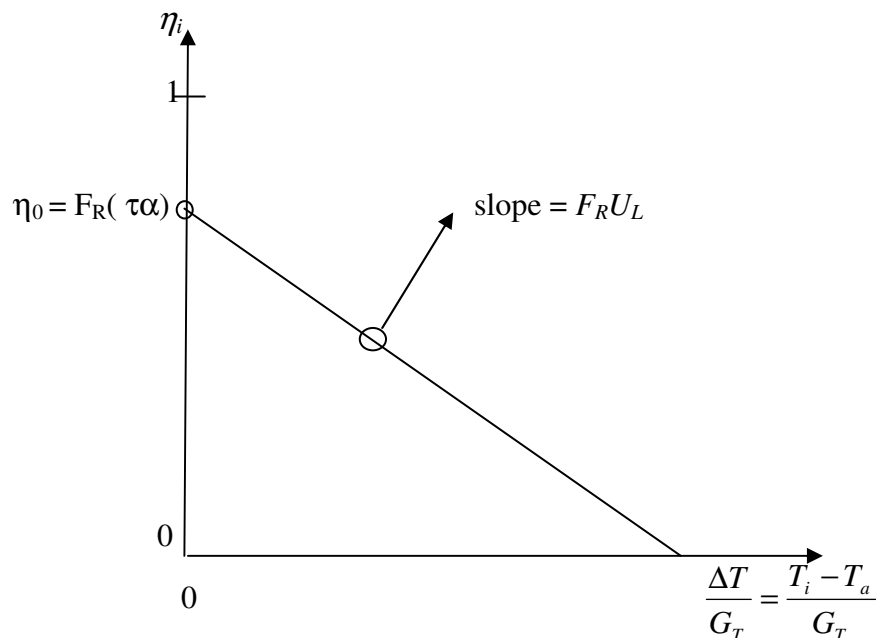


Figure 5.6.1

Exercise 5.6.1

A water heating collector with an aperture area of 4.10 m^2 is tested with the beam radiation nearly normal to the plane of the collector, giving the following information:

Q_u [MJ/hr]	G_T [W/m^2]	T_i [$^{\circ}\text{C}$]	T_a [$^{\circ}\text{C}$]
9.05	864	18.2	10.0
1.98	894	84.1	10.0

What are the $F_R(\tau\alpha)_n$ and $F_R U_L$ for this collector, based upon aperture area? (In practice, tests produce multiple data points and a least squares fit would be used to find the best constants.)

In Europe, another expression is normally used, namely

$$\eta_i = \frac{Q_u}{A_c G_T} = F_{av} (\tau\alpha) - \frac{F_{av} U_L (T_{av} - T_a)}{G_T} \quad (5.6.2)$$

where $T_{av} = (T_o + T_i)/2$ and the relation between F_R and F_{av} is given by

$$F_R = F_{av} \left(1 + \frac{A_c F_{av} U_L}{2\dot{m} C_p} \right)^{-1} \quad (5.6.3)$$

Inserting the incidence angle moderator $K_{\tau\alpha}$ (as defined by Equation 5.5.2) into Equation 5.1.3 changes this into

$$Q_u = A_c F_R [G_T K_{\tau\alpha} (\tau\alpha)_n - U_L (T_i - T_a)] \quad (5.6.4)$$

where

$$K_{\tau\alpha} = 1 + b_o \left(\frac{1}{\cos \theta} - 1 \right) \quad (5.6.5)$$

and the incidence angle modifier coefficient b_o typically is between -0.10 and -0.20.

5.7 Energy Storage: Water Tanks

DB 8.3-4, 10.10

Energy (heat) can be stored in many forms, commonly as heat without phase change (sensible heat) in water.

The useful one-cycle capacity of a water tank is given by

$$Q_s = (mC_p)_s \Delta T_s \quad [\text{J}] \quad (5.7.1)$$

where ΔT_s is the temperature range. For a fully mixed (non-stratified) tank, we get the following power balance

$$(mC_p)_s \frac{dT_s}{dt} = Q_u - L_s - (UA)_s (T_s - T_a') \quad (5.7.2)$$

where the three terms represent the energy input from the collector, the load, and the loss, respectively. Numerical integration of Equation 5.7.2 is done by calculating a new temperature (T_s^+) e. g. once per hour:

$$T_s^+ = T_s + \frac{\Delta t}{(mC_p)_s} [Q_u - L_s - (UA)_s (T_s - T_a')] \quad (5.7.3)$$

A modern solar heating system increasingly frequently includes a stratified water storage tank. Such a tank is arranged so cold water is taken from the tank's bottom to the solar collector loop (via a heat exchanger), and hot water from the collector loop is added at the top or, even better, at the position in the tank where the temperature is the same as the water from the collector.

Such a tank is usually a part of a so-called low-flow system: Cold water enters the solar collector and passes it so slowly that it is quite hot when it gets back to the heat exchanger. Such a system delivers much useful heat, lowers the requirement of auxiliary heat, but is still effective, since the low inlet temperature brings the average absorber temperature (and thus the heat losses) down.

Solar fraction

The solar fraction, i. e. the percentage of the (seasonal or yearly) heat load is increasingly used as a measure of how well a system performs. Recent research at SERC in Sweden has as an example shown that the solar fraction of a typical summer season hot water system can be doubled if the classical high-flow system with a copper tubing coil in the bottom of the storage tank is replaced with a low-flow system with efficient so called SST heat exchanger and a stratified tank, without increasing the collector area.

Chapter 6

Semi Conductors and P-N Junctions

6.1 Semiconductors

(AP 2.1-2)

Valence bands have electrons stuck to the covalent bonds in the lattice. Conduction bands may have free electrons. Between the bands is a forbidden band with band gap E_G . The Fermi level E_F is close to mid-gap in pure semiconductors. Semiconductors have $E_G = 0.4-4$ eV. Insulators have $E_G \gg 4$ eV. Metals have no band gap.

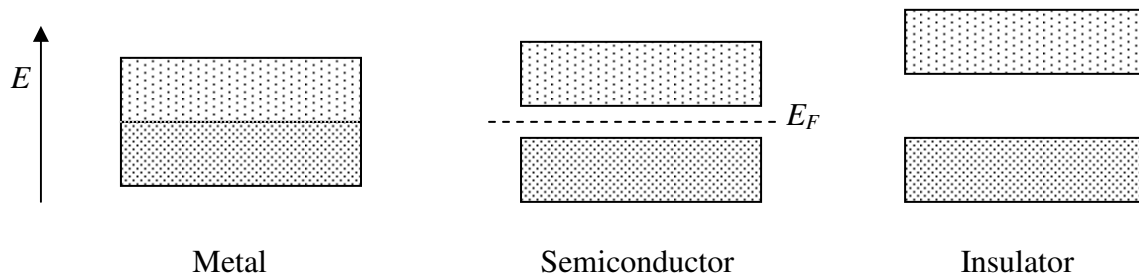


Figure 6.1.1. Bands (bottom to top): Valence band, forbidden band, conduction band.

Semiconductors are (normally) crystals with diamond lattice and each atom surrounded by four atoms. They can be group IV atoms (Si, Ge) or a mixture of group III (Ga, Cu, In) and group V (As, Se, Sb) atoms.

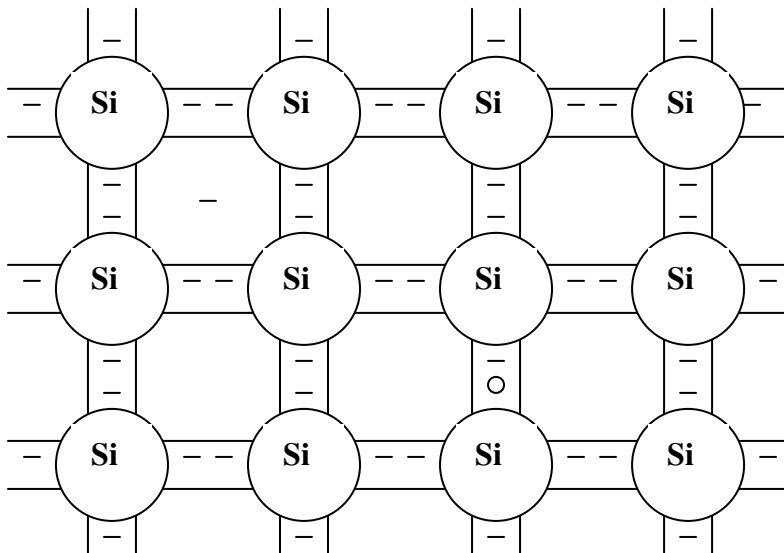


Figure 6.1.2. Two-dimensional analogy of a small part of a Si crystal with a thermally generated electron-hole pair.

Neither a filled valence band nor an empty conduction band can conduct any current. In semiconductors, Fermi-Dirac statistics however permits (at room temperature) a small amount of electrons in the conduction band, leaving holes behind in the valence band. Both carriers may conduct current, thus making the crystal "semiconductive". (Analogy: Parking garage w. first floor totally filled and second floor empty + move one car from first to second floor.)

By doping a semiconductor with Group III or V atoms, free carriers can be created. The energy needed to break the bond of the extra carrier, approx. 0.02 eV, is so low that in principle a free carrier per doping atom is created.

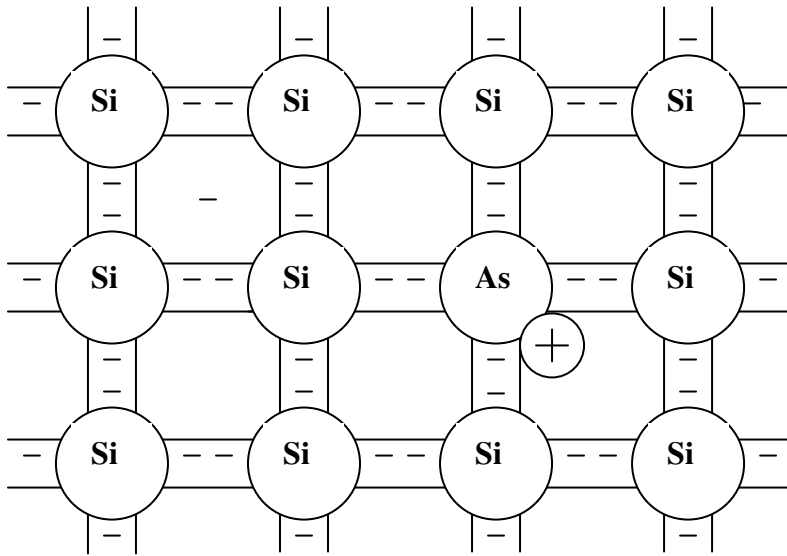


Figure 6.1.3. Small part of Si crystal doped with donor atoms from Group V (As), resulting in free negative carriers (electrons) and bound positive charges.

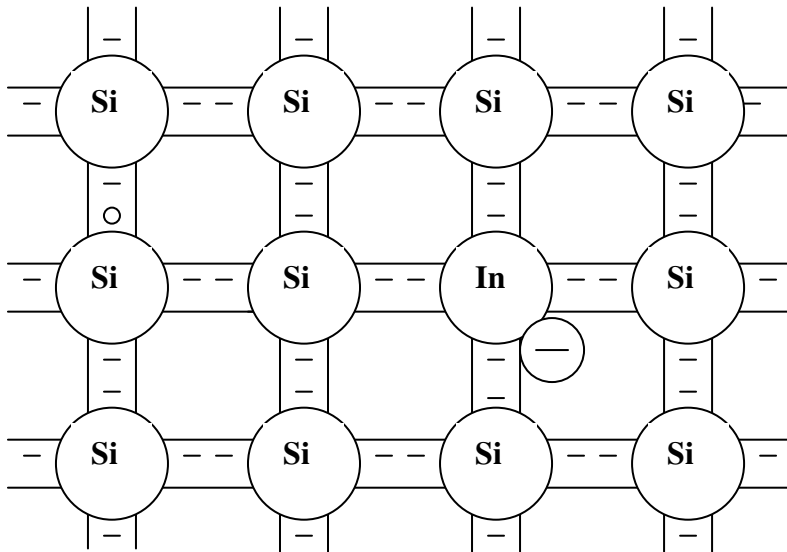


Figure 6.1.4. Small part of Si crystal doped with acceptor atoms from Group III (In), resulting in free positive carriers (holes) and bound negative charges.

More on semiconductors for solar cell applications is found in Section 7.4

6.2 P-N Junctions

(AP 2.5)

Doping a semiconductor with Group III atoms moves the Fermi level close to the valence band; see Figure 6.2.1 (a). (b) Doping a semiconductor with Group V atoms moves the Fermi level close to the conduction band; see Figure 6.2.1 (b).

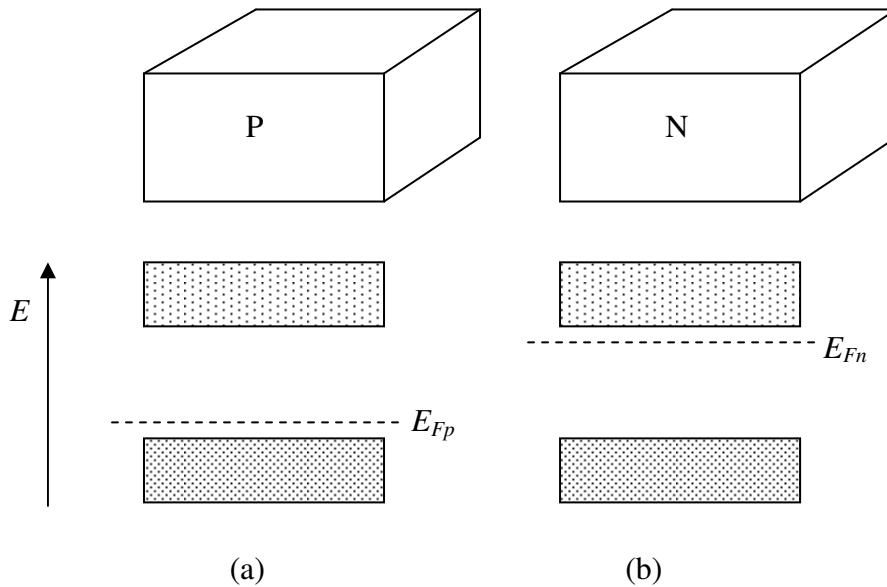


Figure 6.2.1. (a) Semiconductor doped with acceptor atoms.
(b) Semiconductor doped with donor atoms.

A p - n junction is formed by bringing the p -type and n -type regions together in a conceptual experiment. The characteristics of the equilibrium situation can be found by considering Fermi levels: *A system in thermal equilibrium can have only one Fermi level.* See Figure 6.2.2.

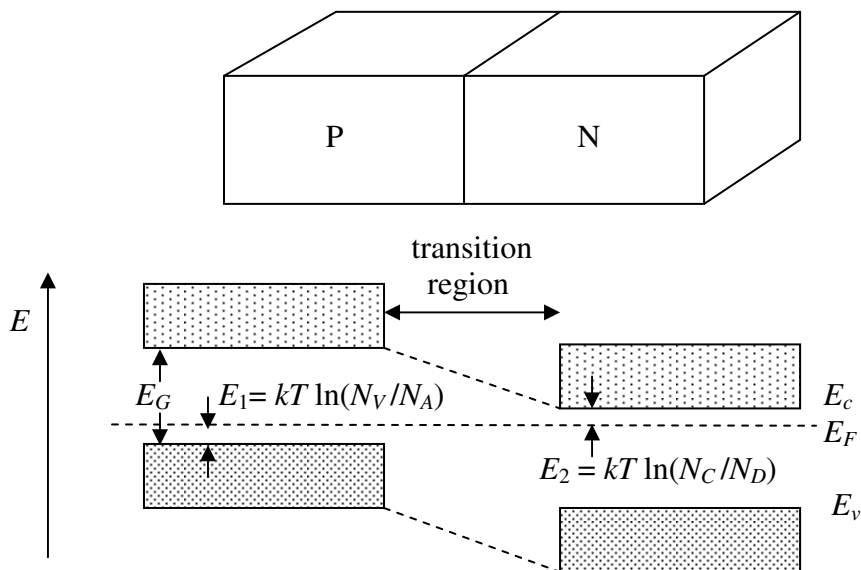


Figure 6.2.2. p - n junction. N_V = effective density of states in the valence band. N_A = density of acceptors. N_C = effective density of states in the conduction band. N_D = density of donors. (For Si, $N_V = 1 \cdot 10^{25} \text{ m}^{-3}$ and $N_C = 3 \cdot 10^{25} \text{ m}^{-3}$.)

When joined, the excess holes in the p -type material flow by diffusion to the n -type, while the electrons flow by diffusion from n -type to p -type as a result of the carrier concentration gradients across the junction. They leave behind exposed charges on dopant atom sites, fixed in the crystal lattice. An electric field \hat{E} therefore builds up in the so-called "depletion region" around the junction to stop the flow.

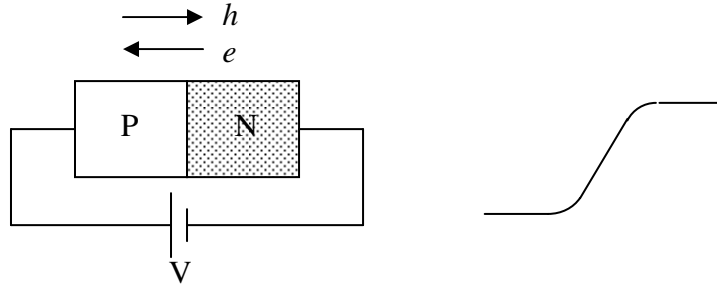


Figure 6.2.3. (a) Application of a voltage to a p - n junction. (b) Shape of electric field.

If a voltage is applied to the junction as shown in Figure 6.2.3, \hat{E} will be reduced and a current flows through the diode (if the external voltage is large enough). If a reverse voltage is applied, no current will flow (or, rather, a low current I_0 due to thermally generated electron-hole pairs). This gives the diode law:

$$I = I_0[\exp(qV/nkT) - 1] \quad (6.2.1)$$

where I_0 is called "dark saturation current", q = electric unit charge, k = Boltzmann's constant, T = absolute temperature, and n = the ideality factor, a number between 1 and 2 which typically increases when the current decreases. **Note** that $kT/q = 0.026$ V for $T = 300$ K.

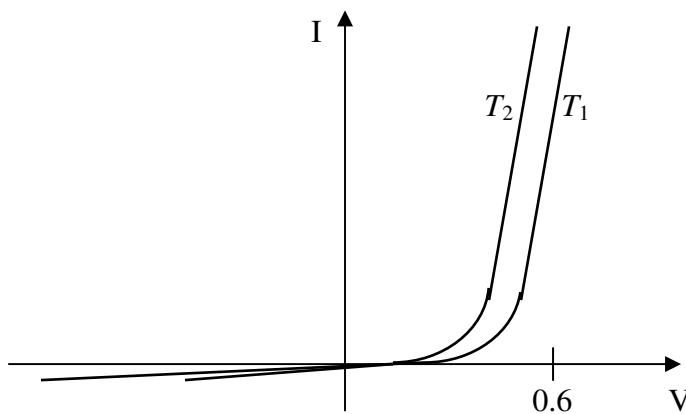


Figure 6.2.4. The diode law for silicon. $T_2 > T_1$. The temperature shift is approx. 2 mV/°C.

Exercise 6.2.1

$I_0 = 5.0 \cdot 10^{-12}$ A for a diode. Draw an I - V characteristics for this diode. What is V for $I = 2.0$ A? Assume $T = 300$ K and $n = 1$.

Chapter 7

The Behavior of Solar Cells

7.1 Absorption of light

(AP 2.3-5)

Fundamental absorption = the annihilation of photons by the excitation of an electron from the valence band up to the conduction band, leaving a hole behind. The energy difference between the initial and the final state is equal to the photon's energy:

$$E_f - E_i = h\nu \quad (7.1.1)$$

For a free particle, kinetic energy and momentum are related by

$$E_k [= mv^2/2 = (mv)^2/2m] = p^2/2m \quad (7.1.2)$$

Similarly for electrons in the conduction band E_c and holes in the valence band E_v :

$$E - E_c = p^2/2m_e^* \quad \text{and} \quad E_v - E = p^2/2m_h^* \quad (7.1.3 \text{ a, b})$$

where p is known as the crystal momentum, and m_e^* and m_h^* are the "effective masses" of the carriers in the lattice. From Equation (7.1.3) is seen both that the conduction band has its minimum and the valence band its maximum for $p = 0$; this is called a *direct-band-gap* device like *GaAs*. *Si* and *Ge* are however examples of *indirect-band-gap* devices where the conduction band minimum occurs at a finite momentum p_0 . Then Equation (7.1.3 a) is replaced by

$$E - E_c = (p - p_0)^2/2m_e^* \quad (7.1.4)$$

The situations are presented in Figure 7.1.1:

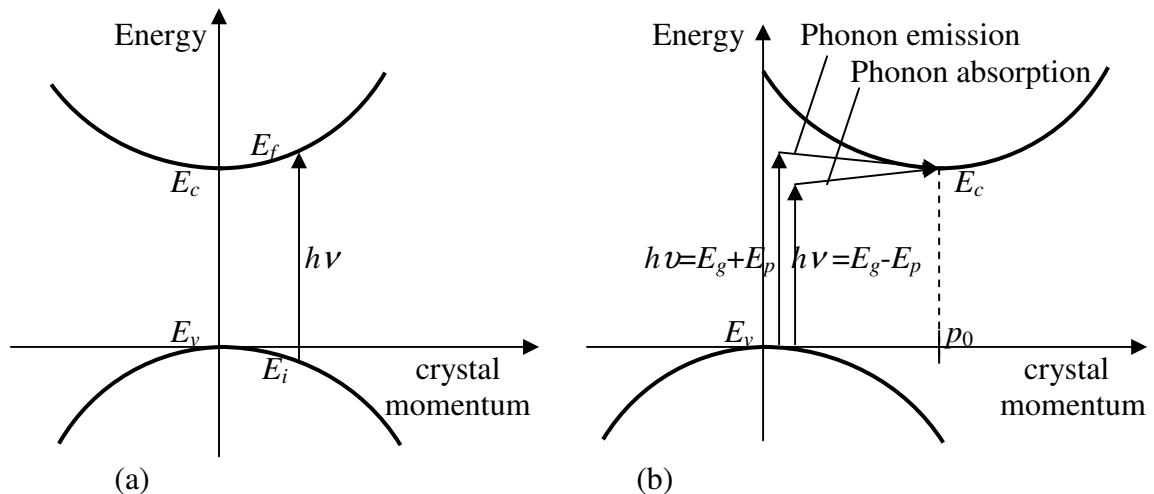


Figure 7.1.1. Energy-crystal relationships near the band edges for electrons in the conduction band and holes in the valence band of (a) a direct-band-gap device, (b) an indirect-band-gap device.

For transitions, see text.

In a direct-band-gap semiconductor, a photon is readily absorbed within a few μm if its energy $hf > E_g = E_c - E_v$. In an indirect-band-gap semiconductor, a phonon (= a quantum corresponding to coordinated vibration in the crystal structure) must be absorbed or emitted (a phonon has high momentum in comparison with photons). Therefore, if we want low-energy photons of the near-infrared absorbed, a device thickness of 100 μm or more is required. These are the energy limits in crystalline *Si* for phonon-absorption process, phonon-emission process, and direct process (at $T = 0$ K), respectively:

$$\begin{aligned} E_{g1}(0) &= 1.1557 \text{ eV} \\ E_{g2}(0) &= 2.5 \text{ eV} \\ E_{gd}(0) &= 3.2 \text{ eV} \end{aligned}$$

At room temperature, E_g for *Si* is approximated to 1.1 eV.

A useful relation for photons: $\lambda [\mu\text{m}] \times hf [\text{eV}] = 1.24$.

When an absorbed photon has lower energy than E_g , no electron is lifted from the valence band to the conduction band, and all its energy is dissipated as thermal energy in the device. When a photon has higher energy than E_g , the remaining energy of an absorbed photon is dissipated as thermal energy in the device, and thus lost. The fraction of the photon's energy that is available for the process is therefore:

$$\begin{aligned} E_{\text{useful}}/hf &= 0 && \text{for } hf < E_g && \text{and} \\ E_{\text{useful}}/hf &= E_g/hf && \text{for } hf > E_g \end{aligned} \quad (7.1.5)$$

The number of electrons lifted into the conduction band per incident photon is called the *quantum efficiency* Q_E . Ideally, the internal $Q_E = 1$ for $hf > E_g$ (and = 0 for $hf < E_g$). The external Q_E includes the transmittance τ of the surface of the device. For a non-treated surface, τ can be rather low. Surface reflectance ρ ($= 1 - \tau$) for normal incidence is given by the equation

$$\rho = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2} \quad (7.1.6)$$

where the (complex) index of refraction of a non-transparent dielectric, $n_c = n - ik$, where k is known as the *extinction coefficient*. As an example, k is very low for *Si* and $hf < 3$ eV, while n is about 3.5 throughout most of the solar spectrum.

Exercise 7.1.1

Prove that anti-reflective coating of *Si* solar cells is necessary by calculating the normal incidence transmittance of a non-treated *Si* surface.

Note: Solar cell \equiv photovoltaic cell \equiv PV cell. A PV module consists of a number of PV cells, usually connected in series. A PV panel (or solar panel) consists of one or more PV modules, connected in parallel or in series (or a combination of both). Modules and panels will be discussed in the following chapters.

7.2 Effect of light

(AP 3.1-3)

The photovoltaic effect is the combination of photoelectric effect (that creates free carriers) and the diode properties, a potential jump over the barrier from the P zone to the N zone. Figure 7.2.1 illustrates this.

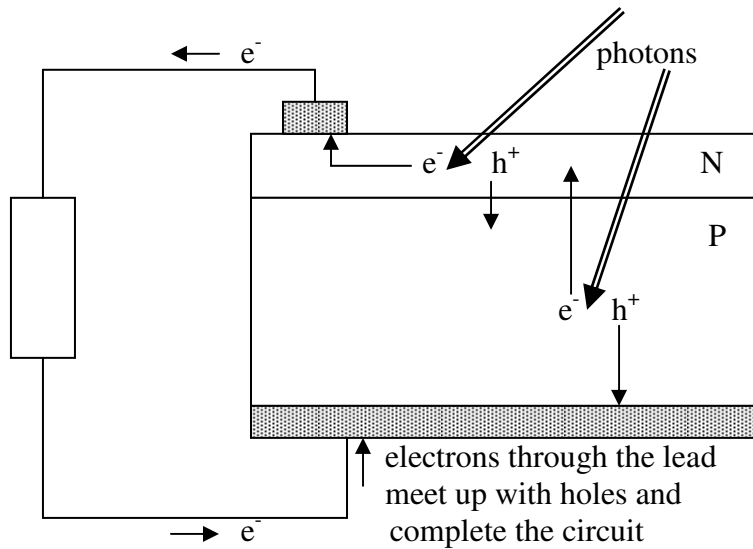


Figure 7.2.1. Creation of electron-hole pairs by the photoelectric effect and flow of electrons and holes at the p - n junction.

A solar cell is a p - n diode with a large surface (typically 1 dm^2) exposed to sunlight. The light-generated current I_L (directly proportional to insolation G) has to be included into the diode law (see Figure 7.2.2):

$$I = I_0[\exp(qV/nkT) - 1] - I_L \quad (7.2.1)$$

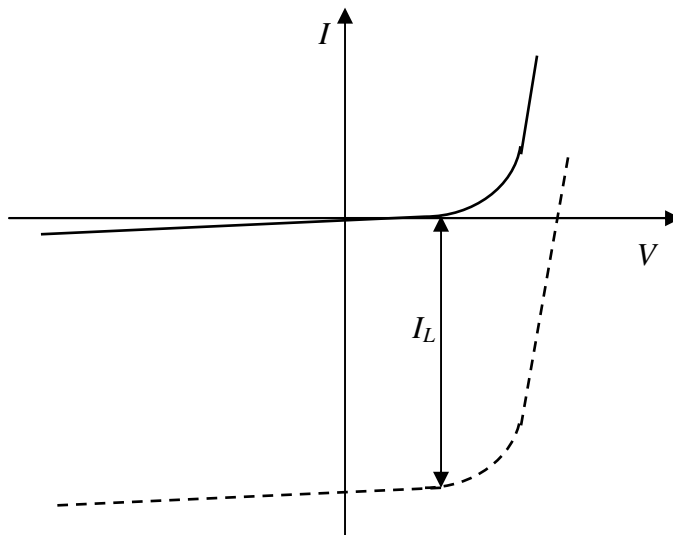


Figure 7.2.2. The effect of light on the I - V characteristics of a p - n junction.

This curve is most often shown reversed with the output curve in the first quadrant, and represented by:

$$I = I_L - I_0[\exp(qV/nkT) - 1] \quad (7.2.2)$$

Four parameters are used to characterize the output of solar cells for given irradiance and area: short-circuit current I_{sc} , open-circuit voltage V_{oc} , fill factor FF , and operating temperature T [K] (or t [°C]).

I_{sc} is the maximum current at zero voltage. Ideally, $V = 0$ gives $I_{sc} = I_L$.

V_{oc} is the maximum voltage at zero current. Setting $I = 0$ in Equation (7.2.2) gives

$$V_{oc} = \frac{nkT}{q} \ln\left(\frac{I_L}{I_0} + 1\right) \quad (7.2.3)$$

The open circuit voltage is thus a function of T , I_L and I_0 .

The diode saturation current I_0 is related to the band gap E_g . A reasonable estimate is given by

$$I_0 = 1.5 \cdot 10^9 \exp\left(-\frac{E_g}{kT}\right) \cdot A \quad [\text{A}] \quad (7.2.4)$$

where $A = \text{cell area [m}^2\text{]}$.

Exercise 7.2.1

Calculate I_0 and V_{oc} for a 1 dm² Si cell with $I_{sc} = 3.0$ A. Assume ideality factor $n = 1$.

FF is a measure of the junction quality and the series resistance (see Sect. 7.3), and is defined by

$$FF = \frac{V_{mp} I_{mp}}{V_{oc} I_{sc}} \quad (7.2.5)$$

where V_{mp} and I_{mp} are the voltage and current at the point of the IV -curve that gives maximum output power P_m . It follows that

$$P_m = V_{mp} \cdot I_{mp} = V_{oc} \cdot I_{sc} \cdot FF \quad (7.2.6)$$

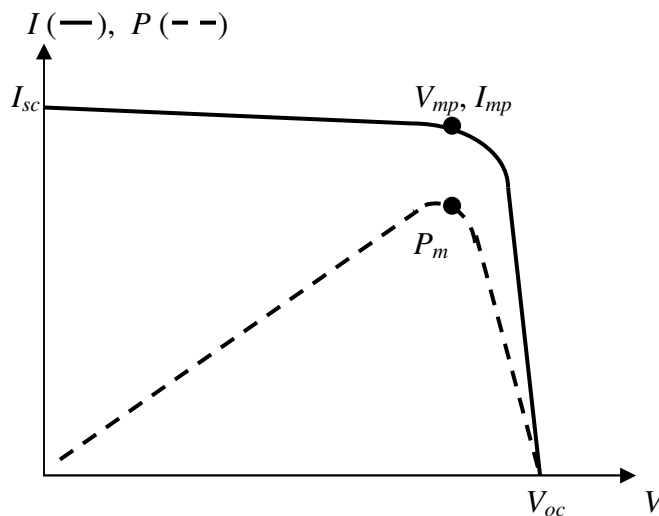


Figure 7.2.3. Typical I - V characteristics of a photovoltaic cell.

Temperature affects the other parameters. Increased temperature causes both decreased E_g and increased I_0 . A lower band gap (usually) implies a higher I_{sc} since more photons have enough energy to create n - p pairs, but this is a small effect. For Si ,

$$dI_{sc}/dT \approx +I_{sc} \cdot 6 \cdot 10^{-4} \text{ [A/K]} \quad (7.2.7)$$

Lower E_g means lower V_{oc} , but also increased I_0 gives lower V_{oc} (see Equation 7.2.3). The combined effect is (for Si) approximated by

$$dV_{oc}/dT \approx -V_{oc} \cdot 3 \cdot 10^{-3} \text{ [V/K]} \quad (7.2.8)$$

Also the fill factor is lowered with increased temperature. For Si ,

$$d(FF)/dT \approx -FF \cdot 1.5 \cdot 10^{-3} \text{ [K}^{-1}\text{]} \quad (7.2.9)$$

Exercise 7.2.2

P_m for a specific Si cell is 1.50 W at 20°C (and $G = 1000 \text{ W/m}^2$). What is P_m for this cell at 60°C?

7.3 One-diode model of PV cell

(AP 3.4)

The one-diode model of a PV cell is shown in Figure 7.3.1:

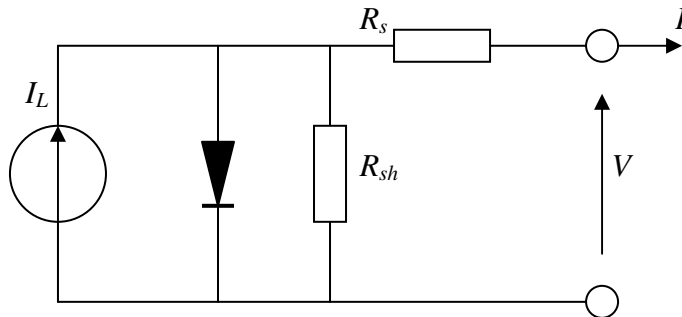


Figure 7.3.1. One-diode model of PV cell with parasitic series and shunt resistances.

The major contributors to the series resistance R_s are the bulk resistance of the semiconductor material, the metallic contacts and interconnections, and the contact resistance between the metallic contacts and the semiconductor. The shunt resistance R_{sh} is due to p - n junction non-idealities and impurities near the junction, which causes partial shortening of the junction, particularly near the cell edges.

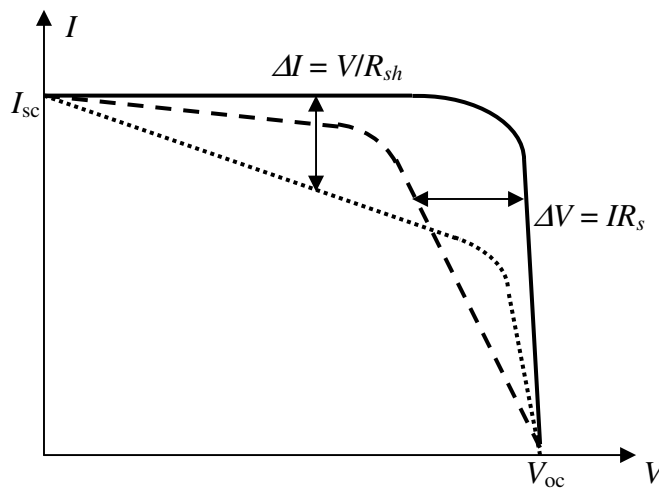


Figure 7.3.2. The effect of series (- - -) and shunt (.....) resistances.

In presence of both series and shunt resistances, the I - V curve of the PV cell is given by

$$I = I_L - I_0 \left\{ \exp \left[\frac{V + IR_s}{(nkT/q)} \right] - 1 \right\} - \frac{V + IR_s}{R_{sh}} \quad (7.3.1)$$

Exercise 7.3.1

(a) When a cell temperature is 300 K, a certain silicon cell of 1 dm^2 area gives an open circuit voltage of 600 mV and a short circuit current output of 3.3 A under 1 kW/m^2 illumination. Assuming that the cell behaves ideally, what is its energy conversion efficiency at the maximum power point? (b) What would be its corresponding efficiency if the cell had a series resistance of $0.1 \text{ } \Omega$ and a shunt resistance of $3 \text{ } \Omega$?

7.4 Cell properties

(AP 2.2, 4.1-3)

Solar cells are most readily commercially available mounted together in modules. The following are available:

* *X-Si* cells, mono- or polycrystalline, circular (dia. 5-10 cm) or square (5×5 - 15×15 cm) with efficiencies $\eta = P_{out}/G$ between 12 and 18 %. Typical thickness $\geq 100 \mu\text{m}$. $E_g = 1.1 \text{ eV}$. *Ge* cells with $E_g = 0.7 \text{ eV}$ are available as sensors only.

* *A-Si:H* cells of amorphous *Si* with hydrogen atoms attached to dangling bonds come in all module sizes from a cm^2 to a m^2 or more. These are thin-film cells of thickness a few μm , produced by diffusion onto a substrate (usually glass or plastic). $\eta = 5\text{-}10 \%$. $E_g = 1.7 \text{ eV}$.

* *III-V* thin film cells are gradually being marketed: *GaAs* cells have a band gap of 1.4 eV, *CdS* cells 2.5 eV, *GaSb* cells 0.7 eV, and $\text{In}_x\text{Ga}_{1-x}\text{As}$ a band gap between 0.4 and 1.3 eV depending of the relative fractions of *In* (x) and *Ga* ($1-x$). *CIS* cells (CuInSe_2) are developed at, among other laboratories, the Ångström Laboratory in Uppsala. Efficiencies vary between 10 and 20 % (with "laboratory best" over 30 %).

* Tandem cells are also becoming available. A double *A-Si* cell will have doubled output voltage. Since the monochromatic efficiency decreases rapidly with decreased wavelength below that corresponding to the band gap, tandem cells with high band gap cells over low band gap cells increase efficiency over that of single cells. (Low band gap cells are also useful as single cells in TPV - thermo photovoltaic - applications where radiation from an emitter at about 1000°C is converted into electricity.)

Exercise 7.4.1

Calculate the wavelengths for which the different kinds of cells mentioned above can convert light into electricity. Explain (qualitatively) why there is an optimum band gap (which happens to coincide well with that of *X-Si*) for conversion of sunlight.

In order to make high-efficiency solar cells, losses have to be minimized. They are of two kinds: Optical losses and recombination losses.

Optical losses occur by reflection of the metal grid on the surface, the reflectivity of the surface, and the transparency of the cell. Surface reflectivity is minimized by anti reflective coating, a transparent coating of thickness $\lambda/4$ and refractive index $= \sqrt{n}$ (where n = the refractive index of the cell material). Such a coating can bring reflectivity down to zero for one wavelength only but can reduce the overall loss of sunlight to 1/2. The losses due to transparency are minimized if the optical path within the cell is long enough, which is achieved with reflective backing and texturing of the front and back surfaces.

Recombination can occur via three different mechanisms: Radiative recombination, which is the reverse of fundamental absorption. Auger recombination, when an electron recombining with a hole gives its energy to an electron that moves within a band and then relaxes back to its original energy state, releasing phonons. Recombination through traps, when electrons recombine with holes in a two stage process, first relaxing to a defect energy level within the forbidden gap, then to the valence band.

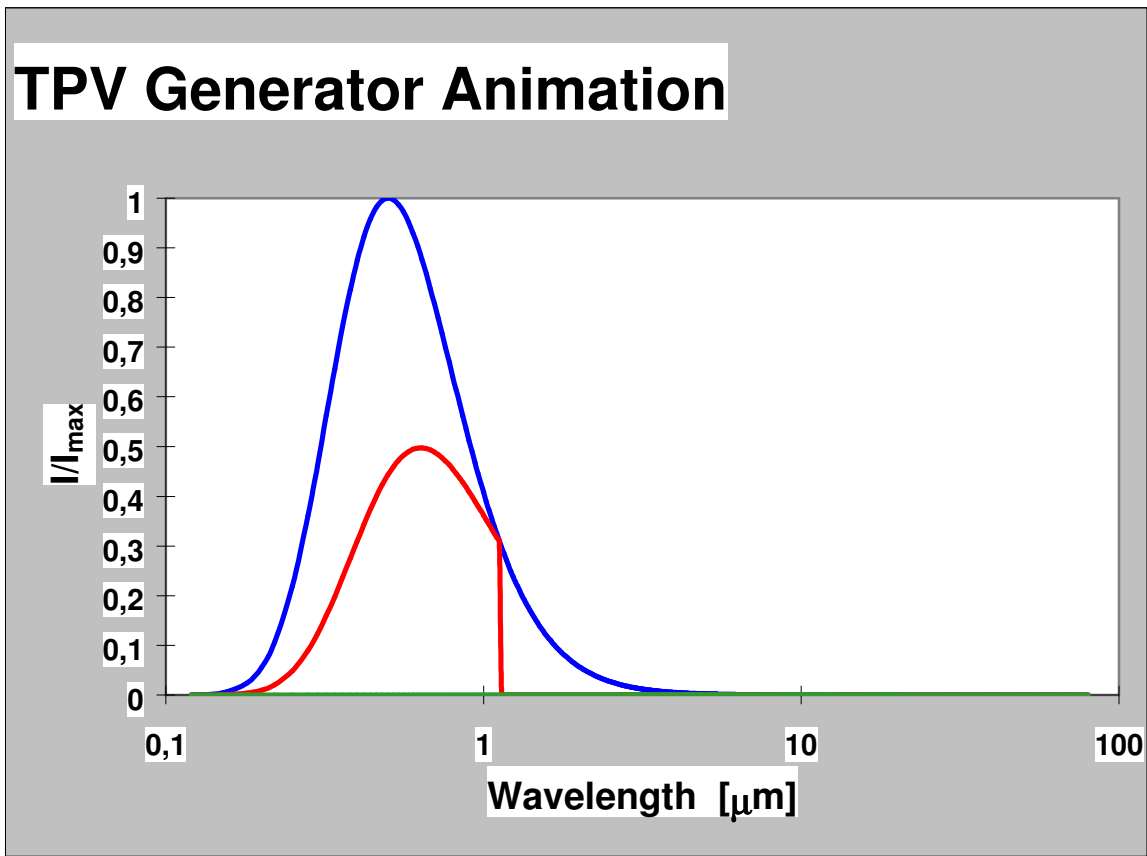


Figure 7.4.1. The lower curve indicates how much of the solar spectrum that is available for PV-generated electricity in a device with band gap 1.1 eV. The animator is available at www.du.se/tpv and can among other things show this availability for any device band gap.

Chapter 8

Stand-Alone Photovoltaic Systems

8.1 Design and modules

(AP 6.1-3)

Photovoltaic cells and systems have a wide and increasing variety of applications, including satellites, navigational aids, telecommunication, small consumer products, battery charging (in boats, caravans, and cabins), developing country applications (light, refrigeration, water pumping), solar powered vehicles, and residential power where there is no grid available. The number of PV panels on grid-connected houses in the world is increasing, but requires (for the time being) governmental support.

For households (and applications of similar size), we distinguish between stand-alone and grid connected systems. Stand-alone systems need a back-up storage, usually lead-acid batteries. A typical such system is shown in Figure 8.1.1.

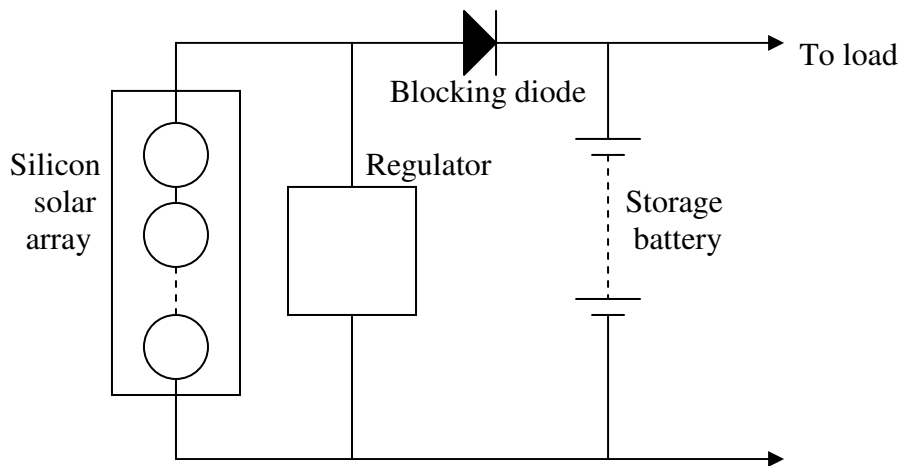


Figure 8.1.1. Simplified stand-alone PV power system.

Typical characteristics for (a) each cell, and (b) a module with 36 cells in series at $G = 1 \text{ kW/m}^2$:

	(a)	(b)
V_{oc}	600 mV (at 25°C)	21.6 V (at 25°C)
I_{sc}	3.0 A	3.0 A
V_{mp}	500 mV (at 25°C)	18 V (at 25°C)
I_{mp}	2.7 A	2.7 A
Area	1 dm ²	0.4×0.9 m

In practice, encapsulated cells usually have lower average efficiencies than unencapsulated cells due to (i) reflection from the glass; (ii) change in reflection from encapsulant/cell interface; (iii) mismatch between cells; and (iv) resistive losses in inter-connects.

A module V_{mp} of 18 V is required when charging a 12 V lead-acid battery because (i) approx. 2.8 V is lost when temperature rises to 60°C; (ii) a drop of 0.6 V in the blocking diode; (iii) a drop of 1.0 V loss across the regulator; (iv) some voltage loss with reduced light intensity; and (v) the batteries must be charged to 14-14.5 V to reach their full state of charge.

8.2 Batteries

(AP 6.4-5)

There are several types of batteries potentially useful in stand-alone PV systems. At present, the most commonly used are lead-acid batteries.

Battery efficiency can be characterized in three different ways:

- (i) **Coulombic or charge efficiency** = amount of charge [Ah] that can be retrieved divided by the amount of charge put in during charging. Typically 85 %.
- (ii) **Voltage efficiency** = the lower voltage when charge is retrieved divided by the higher voltage necessary to put the charge into the battery. Also typically 85 %.
- (iii) **Energy efficiency** = the product of coulombic and voltage efficiencies. Typically 72 %.

Battery capacity is the maximum amount of energy that can be extracted from a battery without the battery voltage falling below a prescribed value; it is given in kWh or Ah at a constant discharge rate. Note that 300 h discharge rate doubles the capacity as compared to 10 h discharge rate for a lead-acid battery. Capacity decreases 1 %/°C below 20°C. Deep cycling batteries can be discharged up to 80 % of rated capacity (but car batteries only 25 %).

Lead-acid batteries are either sealed or open; the latter needs less stringent charging regime but gasses hydrogen (they are also less expensive). The plates are either pure lead (low self-discharge and long life expectancy but soft and easily damaged); lead with calcium added (low initial cost and stronger, but not as suitable for repeated deep discharging); or lead with antimony added (much cheaper than pure-lead or lead-calcium but shorter lives and higher self-discharge rate: consequently not ideal for use in stand-alone PV applications).

A lead-acid battery should not spend long periods at low states of charge due to risk of sulphation (lead sulfide crystals grow on the battery plates and sulfuric acid concentration decreases). A nominal 12 V battery has six cells coupled in series. Battery cell voltage varies both with state of charge and whether it is being charged or discharged. During discharge, a fully charged battery has 2.05 V (at 10 h discharge rate) to 2.10 V (at 300 h discharge rate) voltage per cell. This is reduced to minimum 1.85 V/cell at some 20 % remaining capacity. When this voltage is reached, the regulator should disconnect the battery from the load.

A lead-acid battery should also not spend long periods at overcharge, which causes gassing leading to loss of electrolyte and shedding of active material from the plates. During charging, battery cell voltage varies from 2.05 V at 20 % capacity to 2.25 V at 90 % capacity and a quick increase to 2.7 V at 100 % capacity. Before this voltage is reached, the regulator should disconnect the battery from the solar panel.

Exercise 8.2.1

The coulombic efficiency of a lead-acid battery is 0.86. With a charging voltage from the PV panel of 14.0 V and a discharging voltage to the load of 11.5 V, what is the energy efficiency?

8.3 Household power systems

(AP 9.1)

Remote area power supply systems in non-grid areas may take on a range of configurations with a number of possible electrical energy generating sources. Present generation options include (i) PV modules; (ii) wind generators; (iii) small hydroelectric generators; (iv) diesel or petrol generators; (v) hybrid systems, comprising one or more of the above.

AC or DC? DC appliances are generally more efficient and do not need an inverter. However, DC wiring is heavier duty, requires special switches and may need specialized personnel for installation. Also, a much smaller range of DC appliances is available.

The following appliances have to be rated with respect to necessity, power requirement, time used, and whether DC is available: Lights, refrigerators/freezers, dishwashers, microwave ovens, home entertainment equipment, and general appliances. In a 1993 Australian analysis, total yearly load of a household's use of electricity (no heat!) varies between 1.6 and 9.9 kWh/day.

Normally, a hybrid system has to be employed. Diesel and gasoline generators have their different advantages and disadvantages. A diesel engine may be used with RME and a gasoline engine converted to ethylene alcohol as a fuel, so both are possible renewable energy engines.

Does a combination of wind and PV give a more even output (over the year) than the one or the other alone? An investigation in northwestern Germany points in the negative direction.

Research at SERC in Borlänge, Sweden, aims at developing a system that uses PV panels and solar collectors on the roof for summer half-year demand of electricity and heat, and a wood powder furnace producing heat and - by means of TPV - electricity during the winter half-year. This combination of technologies could make a household self-supporting with both heat and electricity throughout the year on renewable energy only. See also www.serc.se and www.du.se/tpv.

Chapter 9

Grid Connected Photovoltaic Systems

9.1 Photovoltaic systems in buildings

(AP 10.1-2)

Photovoltaics can be used in grid connected mode in two ways: as arrays installed at the end site, e. g. on roof tops, or as utility scale generating stations.

In a building, PV systems can provide power for a number of functions:

- (i) Architectural - dual purpose: electricity generation and roofing, walls, or windows.
- (ii) Demand-side management - to offset peak time loads.
- (iii) Hybrid energy systems - supplementing other sources for lighting, heat pumps, air conditioners, etc.

As (usual) for a stand-alone system, an inverter is needed, since PV arrays generate DC power. Two main types of inverters can be used to achieve AC power at the voltage used in the main grid: These are: (i) line commutated, where the grid signal is used to synchronize the inverter with the grid, and (ii) self commutated, where the inverter's intrinsic electronics lock the inverter signal with that of the grid.

Issues to be considered when selecting an inverter include (i) efficiency, (ii) safety, (iii) power quality, (iv) compatibility (with the array), and presentation (compliance with relevant electric codes, size, weight, construction and materials, protection against weather conditions, terminals and instrumentation).

On-site storage is not essential for grid-connected systems but can greatly increase their value. Storage can be provided on site, typically via batteries, or at grid level, for instance via (i) pumped hydro; (ii) underground caverns, compressed air; and (iii) batteries, superconductors or hydrogen.

To make much impact on household electricity use, a PV system of about 2 kW_p or about 20 m^2 would be needed. A system rated at $3\text{-}4 \text{ kW}_p$ would supply most household needs. Depending on the house design, a limit of about 7 kW_p or 70 m^2 is normally imposed by the available roof area.

Other issues which need to be addressed for household PV systems include (i) aesthetics; (ii) solar access; (iii) building codes; (iv) insurance issues; (v) maintenance; (vi) impact on utility; and (vii) contract with utility.

9.2 Photovoltaic power plants

(AP 10.4)

Despite the relative ease of installation and cost effectiveness of the small PV systems, much utility interest in PV still centers around the development and testing of central, grid connected PV stations, since most utilities are more familiar with large scale, centralized power supply. The technical and economic issues involved in large, central generating PV plant are these:

(i) **Cell interconnection.** In determining the best way of connecting cells in large systems, the potential losses must be examined. For instance, many parallel cells improve tolerance to open-circuits but not to short-circuits. Optimum system tolerance is achieved with single string source circuits and large number of bypass diodes. Field studies show that in large systems it is better to design the system to be tolerant to cell failures than to replace modules containing failed cells.

(ii) **Protective features.** This includes blocking diodes and overcurrent devices, array arcing (70 V maintains an arc), and grounding (frame grounding, circuit grounding, and ground fault breaker).

(iii) **Islanding.** This is the feature of a grid connected PV system to continue to operate even if the grid shuts down.

Finally, the value of PV generated power can be viewed from several perspectives including (i) **global**, taking into account such issues as use of capital, environmental impact, access to power, etc.;

(ii) **societal**, local impacts, manufacturing, employment, cost of power;

(iii) **individual**, initial cost, reduction in utility bills, independence; and

(iv) **utility**, PV output in relation to demand profiles, impact on capital works, maintenance, etc.

Appendix 1: Blackbody Spectrum

Table 1. Fraction of blackbody radiant energy Δf between previous λT and present λT [$\mu\text{m K}$] for different λT -values (reference Duffie-Beckman).

λT	Δf	λT	Δf	λT	Δf	λT	Δf
1000	0.0003	3700	0.0202	6400	0.0074	9100	0.0028
1100	0.0006	3800	0.0196	6500	0.0071	9200	0.0027
1200	0.0012	3900	0.0190	6600	0.0068	9300	0.0026
1300	0.0022	4000	0.0184	6700	0.0066	9400	0.0025
1400	0.0034	4100	0.0179	6800	0.0064	9500	0.0025
1500	0.0051	4200	0.0173	6900	0.0061	9600	0.0024
1600	0.0069	4300	0.0167	7000	0.0058	9700	0.0023
1700	0.0088	4400	0.0161	7100	0.0056	9800	0.0022
1800	0.0108	4500	0.0155	7200	0.0054	9900	0.0021
1900	0.0128	4600	0.0150	7300	0.0053	10000	0.0021
2000	0.0146	4700	0.0144	7400	0.0051		
2100	0.0163	4800	0.0138	7500	0.0049	11000	0.0177
2200	0.0179	4900	0.0134	7600	0.0047	12000	0.0132
2300	0.0191	5000	0.0128	7700	0.0046	13000	0.0100
2400	0.0202	5100	0.0124	7800	0.0043	14000	0.0078
2500	0.0211	5200	0.0118	7900	0.0042	15000	0.0061
2600	0.0218	5300	0.0114	8000	0.0041	16000	0.0048
2700	0.0222	5400	0.0110	8100	0.0039	17000	0.0039
2800	0.0226	5500	0.0106	8200	0.0038	18000	0.0031
2900	0.0227	5600	0.0101	8300	0.0037	19000	0.0026
3000	0.0226	5700	0.0097	8400	0.0035	20000	0.0022
3100	0.0226	5800	0.0094	8500	0.0034	30000	0.0097
3200	0.0223	5900	0.0090	8600	0.0033	40000	0.0026
3300	0.0220	6000	0.0087	8700	0.0032	50000	0.0010
3400	0.0216	6100	0.0083	8800	0.0031	∞	0.0012
3500	0.0212	6200	0.0080	8900	0.0030		
3600	0.0207	6300	0.0077	9000	0.0029		

Table 2. Fraction of blackbody radiant energy Δf between zero and λT [$\mu\text{m K}$] for even fractional increments (reference Duffie-Beckman).

$f_0-\lambda T$	λT [μmK]	λT at midpoint	$f_0-\lambda T$	λT [μmK]	λT at midpoint
0.05	1880	1660	0.55	4410	4250
0.10	2200	2050	0.60	4740	4570
0.15	2450	2320	0.65	5130	4930
0.20	2680	2560	0.70	5590	5350
0.25	2900	2790	0.75	6150	5850
0.30	3120	3010	0.80	6860	6480
0.35	3350	3230	0.85	7850	7310
0.40	3580	3460	0.90	9380	8510
0.45	3830	3710	0.95	12500	10600
0.50	4110	3970	1.00	∞	16300

Appendix 2

Latitudes ϕ of Swedish Cities with Solar Stations and for Jyväskylä

City	Latitude
Kiruna	67.83
Luleå	65.55
Umeå	63.82
Östersund	63.20
Borlänge	60.48
Uppsala	59.85
Karlstad	59.37
Stockholm	59.35
Norrköping	58.58
Göteborg	57.70
Visby	57.67
Växjö	56.93
Lund	55.72
Jyväskylä	62.23

Appendix 3

Average Monthly Insolation Data for Swedish Cities and Jyväskylä, Finland: Global, Beam, and Diffuse Radiation on Horizontal Surfaces

City:	Ki	Lu	Um	Ös	Bo	Up	Ka	St	No	Gö	Vi	Vä	Lu	Jy
M (kWh)														
Jan.	1	4	5	7	9	9	11	10	12	11	12	11	14	6
Feb.	15	19	23	25	28	26	29	27	29	28	29	29	30	20
March	58	59	64	71	69	67	72	67	70	62	74	63	65	66
April	111	108	111	116	99	105	113	107	107	102	119	105	109	105
May	152	153	157	158	156	157	161	162	158	149	176	145	156	151
June	158	172	181	173	169	174	183	176	174	167	190	157	165	150
July	143	161	170	158	159	158	173	160	165	153	178	144	155	154
Aug.	99	111	121	119	123	123	134	126	129	122	137	123	129	121
Sept.	54	59	67	65	70	72	79	76	77	78	84	73	80	73
Oct.	21	24	29	29	33	35	36	37	38	37	42	37	42	23
Nov.	4	6	9	9	12	12	14	14	15	15	15	15	17	7
Dec.	0	1	3	3	6	6	7	7	8	8	8	8	10	3
Mb (kWh)														
Jan.	1	2	2	4	4	3	5	4	5	3	4	3	5	2
Feb.	9	11	13	15	15	13	15	13	14	13	14	13	13	8
March	36	34	37	44	38	36	40	35	37	29	40	29	30	38
April	69	64	65	70	50	56	63	57	57	51	68	53	56	62
May	87	87	91	92	88	89	93	94	90	80	109	76	87	95
June	90	99	108	100	96	101	111	103	101	93	118	83	91	91
July	72	90	99	87	87	86	101	88	93	80	106	71	82	93
Aug.	47	57	66	63	66	65	76	68	71	63	78	63	69	75
Sept.	25	28	34	31	34	35	41	38	39	39	45	33	39	44
Oct.	10	11	14	14	15	16	17	18	18	16	21	15	19	8
Nov.	2	2	4	3	3	3	4	4	4	4	4	3	4	2
Dec.	0	1	2	1	2	2	3	3	3	2	2	2	3	1
Md (kWh)														
Jan.	0	2	3	3	5	6	6	6	7	8	8	8	9	4
Feb.	6	8	10	10	13	13	14	14	15	15	15	16	17	12
March	22	25	27	27	31	31	32	32	33	33	34	34	35	28
April	42	44	46	46	49	49	50	50	50	51	51	52	53	43
May	65	66	66	66	68	68	68	68	68	69	67	69	69	56
June	68	73	73	73	73	73	72	73	73	74	72	74	74	59
July	71	71	71	71	72	72	72	72	72	73	72	73	73	61
Aug.	52	54	55	56	57	58	58	58	58	59	59	60	60	46
Sept.	29	31	33	34	36	37	38	38	38	39	39	40	41	29
Oct.	11	13	15	15	18	19	19	19	20	21	21	22	23	15
Nov.	2	4	5	6	9	9	10	10	11	11	11	12	13	5
Dec.	0	0	1	2	4	4	4	4	5	6	6	6	7	2

Appendix 4

Monthly Average Days, Dates and Declinations

Month	Date	Day of year n	Sun's declination δ
January	17	17	-20.9
February	16	47	-13.0
March	16	75	-2.4
April	15	105	9.4
May	15	135	18.8
June	11	162	23.1
July	17	198	21.2
August	16	228	13.5
September	15	258	2.2
October	15	288	-9.6
November	14	318	-18.9
December	10	344	-23.0

Appendix 5

Spectral Distribution of Terrestrial Beam Radiation at AM 2 (and 23 km Visibility), in Equal Increments, 10 %, of Energy

Energy band	Wavelength Range [nm]	Midpoint Wavelength [nm]
0.0-0.1	300-479	434
0.1-0.2	479-557	517
0.2-0.3	557-633	595
0.3-0.4	633-710	670
0.4-0.5	710-799	752
0.5-0.6	799-894	845
0.6-0.7	894-1035	975
0.7-0.8	1035-1212	1101
0.8-0.9	1212-1603	1310
0.9-1.0	1603-5000	2049

Ref. Wiebelt, J. A. and Henderson, J. B. *ASME J. Heat Transfer* 101, 101 (1979).

Appendix 6

(reference Duffie-Beckman)

(a) Properties of Air at One Atmosphere

T [°C]	ρ [kg/m ³]	C_p [J/kg·K]	k [W/m·K]	$\mu \cdot 10^5$ [Pa·s]	$\alpha \cdot 10^5$ [m ² /s]	Pr
0	1.292	1006	0.0242	1.72	1.86	0.72
20	1.204	1006	0.0257	1.81	2.12	0.71
40	1.127	1007	0.0272	1.90	2.40	0.70
60	1.059	1008	0.0287	1.99	2.69	0.70
80	0.999	1010	0.0302	2.09	3.00	0.70
100	0.946	1012	0.0318	2.18	3.32	0.69
120	0.898	1014	0.0333	2.27	3.66	0.69
140	0.854	1016	0.0345	2.34	3.98	0.69
160	0.815	1019	0.0359	2.42	4.32	0.69
180	0.779	1022	0.0372	2.50	4.67	0.69
200	0.746	1025	0.0386	2.57	5.05	0.68

(b) Properties of Materials

Density ρ [kg/m³]

Copper	8795
Steel	7850
Aluminum	2675
Glass	2515
Water 20°C	1001
Wood	570
Mineral wool	32
Polyurethane foam	24

Specific heat C_p [J/kg·K]

Steel	500
Glass	820
Concrete	840
Water 20°C	4182
Water 40°C	4178
Water 60°C	4184
Water 80°C	4196
Water 100°C	4216

Thermal Conductivity k [W/m·K]

Copper	385
Aluminum	211
Concrete	1.73
Glass	1.05
Water 20°C	0.596
Wood	0.138
Mineral wool	0.034
Polyurethane foam	0.024

(c) Selected Constants

Boltzmann's constant $k = 1.38 \cdot 10^{-23}$ [J/K]

Electric unit charge $q = 1.602 \cdot 10^{-19}$ [As]

Planck's constant $h = 6.6256 \cdot 10^{-34}$ [Js]

Stefan-Boltzmann's constant $\sigma = 5.67 \cdot 10^{-8}$ [W/m²K⁴]

$kT/q = 0.026$ V for $T = 300$ K

Appendix 7 Algorithms for calculating monthly insolation on an arbitrarily tilted surface

The angle of incidence of beam radiation on a surface, θ , is a function of five variables: declination δ , latitude ϕ , surface tilt β , surface azimuth γ and the sun's hour angle ω .

Here we regard θ as a function of ω only, and call the other variables parameters. The general expression for θ can then be written

$$\begin{aligned} f(\omega) = \cos \theta = & (\sin \delta \sin \phi \cos \beta - \sin \delta \cos \phi \sin \beta \cos \gamma) \\ & + (\cos \delta \cos \phi \cos \beta + \cos \delta \sin \phi \sin \beta \cos \gamma) \cos \omega \\ & + (\cos \delta \sin \beta \sin \gamma) \sin \omega \end{aligned} \quad 1.1.1$$

The parameters X , Y , and Z are defined as follows:

$$X = \sin \delta \sin \phi \cos \beta - \sin \delta \cos \phi \sin \beta \cos \gamma \quad 1.1.2$$

$$Y = \cos \delta \cos \phi \cos \beta + \cos \delta \sin \phi \sin \beta \cos \gamma \quad 1.1.3$$

$$Z = \cos \delta \sin \beta \sin \gamma \quad 1.1.4$$

Using these new parameters, $f(\omega)$ becomes

$$f(\omega) = X + Y \cos \omega + Z \sin \omega \quad 1.1.5$$

For a horizontal surface with $\beta = 0$), Equation 1.1.1 becomes

$$g(\omega) = \cos \theta_z = \sin \delta \sin \phi + \cos \delta \cos \phi \cos \omega \quad 1.1.6$$

Defining parameters U and V as

$$U = \sin \delta \sin \phi \quad 1.1.7$$

$$V = \cos \delta \cos \phi \quad 1.1.8$$

simplifies Equation 1.1.6 into

$$g(\omega) = U + V \cos \omega \quad 1.1.9$$

We want to integrate f from "sunrise hour angle" ω_s' to "sunset hour angle" ω_s , which will be defined later. With ω given in radians, a primitive function $F(\omega)$ is

$$F(\omega) = X\omega + Y \sin \omega - Z \cos \omega \quad 1.1.10$$

We want to integrate g from sunrise hour angle ω_s to sunset hour angle ω_s . A primitive function $G(\omega)$ is

$$G(\omega) = U\omega + V \sin \omega \quad 1.1.11$$

In order to determine how an arbitrarily tilted surface behaves, the ratio R_{Mb} between the monthly beam radiation on the surface and that on a horizontal surface is an important number. R_{Mb} equals the ratio between the integral over all incidence angles θ that the surface is (or, rather, can be) lit by beam radiation during the average month day and the same integral for a horizontal surface:

$$R_{Mb} = \frac{\int_{\omega_r'}^{\omega_s'} \cos \theta d\omega}{\int_{\omega_r}^{\omega_s} \cos \theta_z d\omega} \quad 1.1.12$$

The "sunrise hour angle" and "sunset hour angle" refer to the angles when the sun begins and ends to reach the tilted surface. The sun begins to reach the surface when θ or $\theta_z = 0$, whichever happens latest. The sun ends to reach the surface when θ or $\theta_z = 0$, whichever happens first. Calling the two hour angles when $\theta = 0$ ω_t^* and ω_s^* , respectively, we get

$$\omega_t' = \max [\omega_t, \omega_t^*] \quad 1.1.13$$

$$\omega_s' = \min [\omega_s, \omega_s^*] \quad 1.1.14$$

The angles ω_t and ω_s are found by setting $g(\omega) = 0$ in Equation 1.1.9, giving

$$\omega_s = \arccos(-U/V) \quad 1.1.15$$

and

$$\omega_t = -\arccos(-U/V) \quad 1.1.16$$

The angles ω_t^* and ω_s^* are calculated by setting $f(\omega) = 0$ in Equation 1.1.5:

$$Y \cos \omega + Z \sin \omega = -X \quad 1.1.17$$

$$\frac{Y}{\sqrt{Y^2 + Z^2}} \cos \omega + \frac{Z}{\sqrt{Y^2 + Z^2}} \sin \omega = \frac{-X}{\sqrt{Y^2 + Z^2}} \quad 1.1.18$$

which we simpler write

$$Y' \cos \omega + Z' \sin \omega = -X' \quad 1.1.19$$

Now,

$$\cos \psi \cos \omega + \sin \psi \sin \omega = \cos \xi \quad 1.1.20$$

where

$$\psi = \arcsin Z' \quad 1.1.21$$

and

$$\xi = \arccos(-X') \quad 1.1.22$$

Then,

$$\cos(\omega - \psi) = \cos \xi \quad 1.1.23$$

giving

$$\omega_r^* = \psi - \xi \quad 1.1.24$$

and

$$\omega_s^* = \psi + \xi \quad 1.1.25$$

Equation 1.1.12 can now be solved:

$$R_{Mb} = \frac{\int_{\omega_r}^{\omega_s} \cos \theta d\omega}{\int_{\omega_r}^{\omega_s} \cos \theta_z d\omega} = \frac{[F(\omega)]_{\omega_r}^{\omega_s}}{[G(\omega)]_{\omega_r}^{\omega_s}} =$$

$$= \frac{X(\omega_s - \omega_r) + Y(\sin \omega_s - \sin \omega_r) - Z(\cos \omega_s - \cos \omega_r)}{U(\omega_s - \omega_r) + V(\sin \omega_s - \sin \omega_r)} \quad 1.1.26$$

When average M , M_b , and M_d for the location are known, the average monthly insolation on any surface is given by

$$M_T = M_b R_{Mb} + M_d \frac{1 + \cos \beta}{2} + M \rho_g \frac{1 - \cos \beta}{2} \quad 1.1.27$$

An Excel[®] file that calculates R_{Mb} when values for the four parameters δ , ϕ , β , and γ are inserted is available separately.

Appendix 8

Answers to Selected Exercises

1.1.1. 10 %, 46 %, 44 %.

1.2.1. 18.46 kJ/m², day

1.3.1. 48°.

1.3.2. (a) 20:38 and 15:30. (b) 17:59 and 15:30

1.3.3. 7 h 40 m.

1.4.1. 3.19.

1.5.1. 4.91 MJ/m²,day.

1.5.2. 41 kWh/m²,month.

1.7.1. 718 W/m².

1.8.1. $M_d = 8$, $M_b = 6$ [kWh/m²]. (Measured 10 and 4, respectively.)

1.9.1. June: 160 kWh/m².

2.1.1. (a) 0.50 μm. (b) 46 %.

2.5.1. $Q/A = 24.6$ W/m² and $h_r = 1.23$ W/m²,K.

2.6.1. 2.74 W/m²,K.

2.6.2. 2.30 W/m²,K.

2.7.1. (a) 7.7 m. (b) to 7.6 W/m²,K and 13.2 W/m²,K, respectively.

3.1.1. 0.95(4).

3.1.2. 0.42(4).

3.2.1. $\alpha = 0.88$. $\varepsilon(100^\circ\text{C}) = 0.10$. $\varepsilon(500^\circ\text{C}) = 0.20$.

4.1.1. 4.3(4) % at normal incidence and 9.3(3) at $\theta = 60^\circ$.

4.1.2. 0.85 at normal incidence and 0.71 at $\theta = 60^\circ$.

4.2.1. $\tau = 0.81(3)$. $\alpha = 0.10(3)$. $\rho = 0.08(4)$.

4.2.2. 0.74 (0.739).

4.3.1. 2.84 MJ/m².

4.4.1. 111 kWh.

5.6.1. $F_R U_L = 7.62 \text{ W/m}^2, K. F_R (\tau\alpha)_n = 0.78 .$

6.2.1. 0.69 V.

7.1.1. $\rho = 31 \%$ for a Si surface.

7.2.1. $I_0 = 6.4 \cdot 10^{-12} \text{ A. } V_{oc} = 0.70 \text{ V.}$

7.2.2. 1.27 W.

7.3.1. (a) $3.15 \cdot 0.52 = 1.64 \text{ W} \Rightarrow 16.4 \%$.

(b) Max. for $V + 0.1 \cdot I = 0.56$: $2.40 \cdot 0.32 = 0.77 \text{ W} \Rightarrow 7.7 \%$.

7.4.1. X-Si: 1.13 μm . Ge: 1.77 μm . A-Si: 0.73 μm . GaAs: 0.89 μm . CdS: 0.50 μm .
GaSb: 1.77 μm . InGaS: 0.95-3.1 μm .

8.2.1. 71 %.