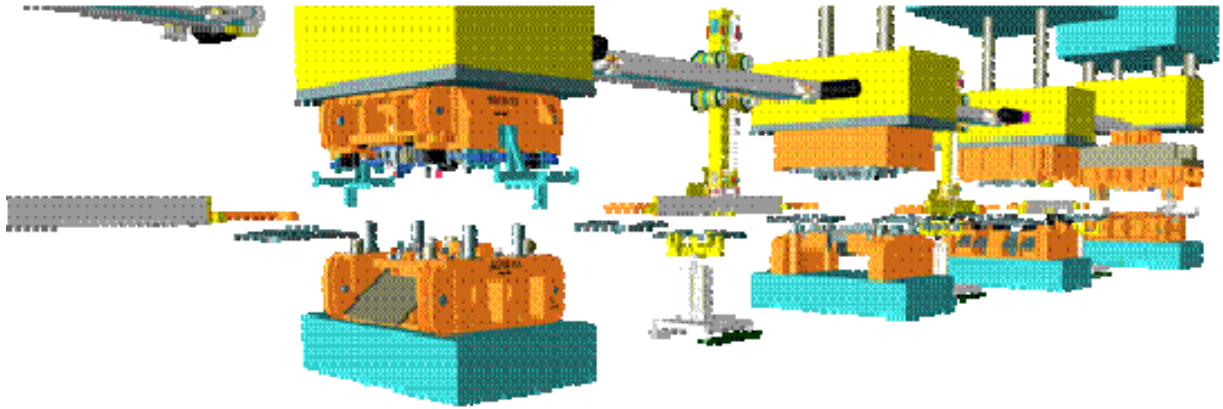


## Appendix C

ENSIL – Mechatronic speciality  
2<sup>nd</sup> year

report of technical studies

# Study of the ‘mix method’



TORRES Stephane

## Appendix C

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## Introduction

For the second part of my project, one of the aims is to test the method developed by Damien RINGENBACH [ref1], so this report is the continuation of Damien's project. This method called the 'mix method' gathers two different algorithms which have been implemented in the Process optimizer. These algorithms look for finding the function optimum. This 'mix method' has been created to take advantages of the two algorithms (Nelder Mead and Direct) in combining them. Direct has the advantage to be very efficient in a global search and Nelder Mead (NM) to be very efficient in a local search. Actually only few tests have been done with this 'mix method' and it seems that it could be an efficient method for the simulation based optimization problem.

### 1 The 'mix method'

The 'mix method' has already been explained [ref 1] but to summarize, it launches a number of Direct iteration, then selects some points in a local search which means that it will select all the local optimum in the points found by Direct or in a global search which means that it will select the global optimum in the points found by Direct. We select these points because we think they are interesting to further investigation. Then there are two options with the selecting points, NM can be launched for all the selecting points or the selecting points can be set in chains and then launch NM for all the chains. The last option is called the gathering process. There is a balance between the Direct algorithm before the selecting process and the Nelder Mead algorithm after the selecting process in order to find the global optimum of the objective function with as few evaluations as possible. Indeed, Direct is normally better to find the interesting areas than NM but NM is normally better than Direct to find the best possible value. So when should we decide if Direct has made enough evaluation to launch the selecting process and then NM for the selecting points is a difficult question.

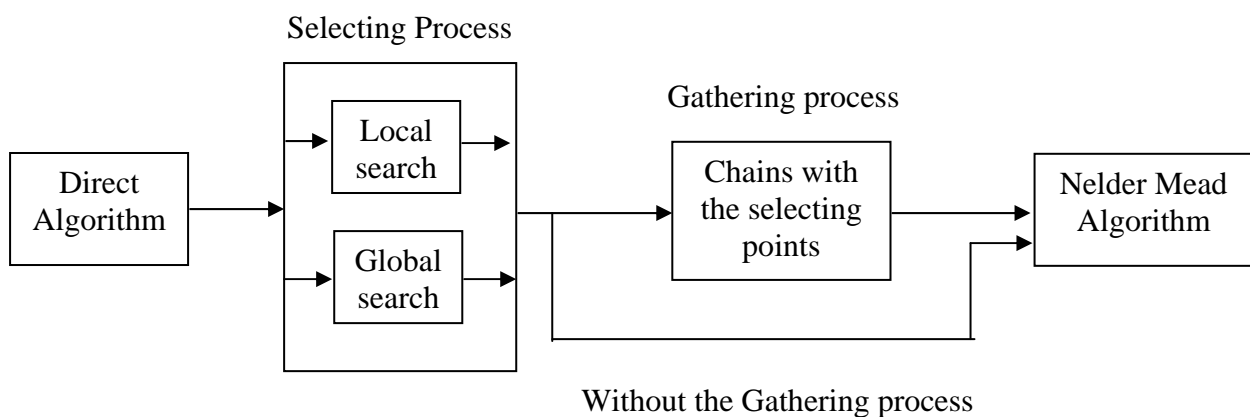


Figure 1 : the 'mix method' scheme

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The selecting process with or without the gathering process and for local or global search is the key part of the 'mix method'. The gathering process uses several parameters like alpha, more alpha is small and more the gathering process will divide chains with the selecting points. Until now, all the tests have been done with  $\alpha=1$  and it will be interesting to see the 'mix method' behaviour for a smaller value, that is to say that the gathering process will create chains but with a distance between the points smaller inside the chain than with  $\alpha=1$  and as there is a criterion, beta, to set the minimal distance between two chains there will be more chains and so new interesting points will be explored. The selecting process selects the points found by Direct, but Direct works only with double whereas the simulation works only with integer. So in the 'mix method' the selecting points are the best Direct points converting in integer. Actually the 'mix method' uses the number total of evaluation predicted to work but this number is just an approximation. This number allows the balance between Direct and NM.

## 2 Number of selected points by different selecting processes

In figure 2, we can see the number of selecting points in a table with all the search type for the 'mix method' and with different alpha values. The first values in this table until the 27 Direct iteration and only for  $\alpha=1$  are the same than in figure 22 [ref 1]. As said in the introduction, the most important part in the 'mix method' is the selecting process but the selecting process may select a lot of points and so it will imply a lot of evaluations to get the results for these points and consequently a lot of consuming simulation time. The idea found by Damien RINGENBACH has been to gather some points which are close each other in chains with a gathering process [ref 1].

In figure 2, we can see that without the gathering process the number of selecting points is bigger than with the gathering process that is the goal. Indeed, with the gathering process the program doesn't need to launch so much NM than without the gathering process. So in a first time, we can say that the gathering process is efficient for the number of selecting points because it reduces this last one.

Without the gathering process, the number of selecting points increases with the Direct iteration to finish at 255 selecting points for the global search and 336 for the local search at the 100<sup>th</sup> direct iteration. This is normal because when the Direct iterations increase the number of points calculated by Direct also increases and so the selecting process selects points among a lot of points. There is an area between almost the 23<sup>rd</sup> Direct iteration and the 50<sup>th</sup> where the selecting process doesn't increase anymore the number of selecting points, and it can even select only one point for the global search (for example at the 23<sup>rd</sup> Direct iteration) or just few points for the local or global search. But after the 50<sup>th</sup> Direct iteration the number of selecting point increases again.

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alpha=1

alpha=0.8

alpha=0.5

Number of iterations	Number total of direct	Before the Gathering		After the Gathering		After the Gathering		After the Gathering	
		Global Search	Local Search	Global Search	Local Search	Global Search	Local Search	Global Search	Local Search
1	21	1	1	1	1	1	1	1	1
2	39	1	1	1	1	1	1	1	1
3	73	1	1	1	1	1	1	1	1
4	103	2	2	2	2	2	2	2	2
5	145	1	3	1	3	1	3	1	3
6	197	3	5	2	4	3	5	3	5
7	257	7	11	5	9	7	11	7	11
8	323	8	9	6	7	8	9	8	9
9	353	11	12	9	10	11	12	11	12
10	407	12	14	10	12	12	14	12	14
11	461	14	18	11	15	14	18	14	18
12	515	16	21	12	17	16	21	16	21
13	571	16	22	12	18	16	22	16	22
14	627	17	23	13	19	17	23	17	23
15	677	17	23	13	19	17	23	17	23
16	727	17	24	13	20	17	24	17	24
17	783	17	24	13	20	17	24	17	24
18	839	17	24	13	20	17	24	17	24
19	895	17	25	13	21	17	25	17	25
20	951	17	25	13	21	17	25	17	25
21	1007	17	26	13	22	17	26	17	26
22	1031	17	26	13	22	17	26	17	26
23	1093	1	1	1	1	1	1	1	1
24	1131	1	1	1	1	1	1	1	1
25	1183	13	13	1	1	1	1	8	8
26	1233	26	26	1	1	1	1	11	11
27	1287	34	34	1	1	1	1	11	11
28	1339	43	43	1	1	1	1	18	18
29	1391	1	39	1	1	1	2	1	16
30	1441	2	45	1	1	1	1	2	23
31	1487	2	45	1	1	1	1	2	23
32	1559	5	5	1	1	1	1	2	2
33	1633	7	7	1	1	1	1	2	2
34	1073	1	7	1	1	1	1	1	2
35	1767	3	20	1	1	1	1	1	2
36	1829	4	20	1	1	1	1	1	2
37	1893	4	20	1	1	1	1	1	2
38	1957	5	21	1	1	1	1	1	2
39	2017	1	21	1	1	1	1	1	2
40	2069	2	26	1	1	1	1	1	2
41	2141	3	27	1	1	1	1	1	2
42	2197	3	27	1	1	1	1	1	2
43	2269	8	15	1	1	1	1	1	2
44	2351	10	26	1	1	1	1	1	2
45	2423	12	31	1	1	1	1	1	2
46	2503	13	34	1	3	1	3	1	3
47	2569	15	36	1	3	1	3	1	4
48	2635	16	37	1	3	1	3	1	4
49	2713	16	37	1	3	1	3	1	4

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50	2781	16	37	1	3	1	3	1	4
51	2847	19	40	1	3	1	3	1	4
52	2913	20	41	1	3	1	3	1	4
53	2979	20	41	1	3	1	3	1	4
54	3045	20	41	1	3	1	3	1	4
55	3109	21	43	1	3	1	3	1	4
56	3173	21	43	1	3	1	3	1	4
57	3237	21	43	1	3	1	3	1	4
58	3303	22	44	1	3	1	3	1	4
59	3355	23	46	1	3	1	4	1	5
60	3401	24	47	1	3	1	4	1	5
61	3445	24	47	1	3	1	4	1	5
62	3489	24	47	1	3	1	4	1	5
63	3535	27	50	1	4	1	4	1	5
64	3581	30	53	1	4	1	4	1	5
65	3627	33	56	1	4	1	4	1	5
66	3673	36	59	1	4	1	4	1	5
67	3719	37	61	1	3	1	4	1	5
68	3765	38	62	1	3	1	4	1	5
69	3811	39	64	1	3	1	4	1	5
70	3865	40	65	1	3	1	4	1	5
71	3909	40	65	1	3	1	4	1	5
72	3953	41	66	1	3	1	4	1	5
73	3997	41	66	1	3	1	4	1	5
74	4041	42	67	1	3	1	4	1	5
75	4095	42	67	1	3	1	4	1	5
76	4139	43	68	1	3	1	4	1	5
77	4197	43	68	1	3	1	4	1	5
78	4241	44	69	1	3	1	4	1	5
79	4281	44	69	1	3	1	4	1	5
80	4321	44	69	1	3	1	4	1	5
81	4389	44	69	1	3	1	4	1	5
82	4445	44	69	1	3	1	4	1	5
83	4493	44	69	1	3	1	4	1	5
84	4539	44	69	1	3	1	4	1	5
85	4585	44	69	1	3	1	4	1	5
86	4629	44	69	1	3	1	4	1	5
87	4689	58	120	1	4	1	5	1	6
88	4747	73	144	1	4	1	5	1	7
89	4815	87	158	1	4	1	5	1	7
90	4875	102	173	1	4	1	5	1	7
91	4935	117	196	1	4	1	5	1	7
92	5017	133	212	1	4	1	5	1	7
93	5077	147	226	1	4	1	5	1	7
94	5135	163	242	1	4	1	5	1	7
95	5193	178	257	1	4	1	5	1	7
96	5261	194	273	1	4	1	5	1	7
97	5329	208	288	1	5	1	6	1	8
98	5395	224	304	1	5	1	6	1	8
99	5463	239	320	1	6	1	7	1	9
100	5529	255	336	1	6	1	7	1	9

Figure 2 : table with the number of points selected for  $\alpha=1$ ,  $\alpha=0.8$  and  $\alpha=0.5$

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The gathering process for  $\alpha=1$  selects less points than the gathering process with  $\alpha=0.8$  or  $0.5$  because the distance between the points inside the chains are smaller with  $\alpha=0.5$  or  $0.8$  than with  $\alpha=1$ . The gathering process for  $\alpha=0.5$  and  $0.8$  selects the same number of selecting points until the 25<sup>th</sup> Direct iteration than without the gathering process for the local search and the global search so it 's not interesting to use  $\alpha < 1$  if we want to use only the 25<sup>th</sup> first Direct iteration. However, after the 25<sup>th</sup> Direct iteration, the gathering process with  $\alpha=0.5$  become interesting. Indeed, it selects only some points from the local and global search without the gathering process, so it reduces the number of NM to launch and that is the goal of the gathering process. But the gathering process seems also interesting for the local and global search with  $\alpha = 1$  and  $0.8$  after the 25<sup>th</sup> Direct iteration because they also select only some points and they select less points than with  $\alpha=0.5$ , so we will lost less time with  $\alpha=1$  and  $0.8$  than with  $\alpha=0.5$  to launch the NM. But among these points, there are a lot of time only point selected.

When the selecting process selects only one point, it means that it selects only the best point found by Direct and all the other points selected without the gathering process are around this best point. In some cases we can see that the gathering process for  $\alpha=1$  or  $0.8$  selects only one point and the gathering process with  $\alpha=0.5$  selects several points, it means that all the points selected without the gathering process are close each other but not so close because with  $\alpha=0.5$  it creates several chains whereas with  $\alpha=0.8$  or  $1$  it creates only 1 chain. We can also see in some cases that the local search selects several points and the global search just one, this can be very good for time simulation consuming to use the global search more than the local search if the NM for the alone point selected provide the same results than all the NM for the several selecting point by the local search.

One thing remarkable is the number of selecting points with the gathering process for the global search which is always 1 after 22<sup>nd</sup> Direct iteration for  $\alpha=1$  and  $0.8$  and after the 33<sup>rd</sup> Direct iteration for  $\alpha=0.5$ . This implies some cases not reachable with the 'mix method' for the global search (see Annex 1 and Annex 2). The gathering process with the local search selects more points than the gathering process with the global search so if the global search has good results for finding the global optimum, it will be preferable to use the global search. After the 32<sup>nd</sup> Direct iteration, the gathering process only select few points, so this is very interesting if in launching NM for these few points it finds the same result than without the gathering process.

To conclude this part, the gathering process is efficient to gather points which are close each other in chains but to be completely efficient it should get the same objective function with the gathering process than without. Alpha is an interesting parameter for the gathering process because if we set  $\alpha=1$ , we will have less selecting points than if we set  $0.8$  or  $0.5$ . But as we get less points with  $\alpha=1$  maybe NM will not find a value as good as with  $\alpha=0.8$  or  $0.5$  where more NM will be launch.



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### 3 Real Number of total evaluation

The 'mix method' uses a parameter called Number total of evaluation predicted which allow the balance between Direct and Nelder Mead (see figure 21 in [ref 1]). This number predicts the amount of the Direct evaluation (already known) and the amount of the Nelder Mead total evaluation (unknown). Until now, the number of evaluation for one Nelder Mead has been predicted with an average of 60 [ref 1] but this average has to be checked. A counter has been implemented in the 'mix method' in order to see the real number of evaluation running by NM. In the figure 3, we can see the real number of evaluation running by the VM for several Direct iteration which is getting with the counter. It gathers the number of Direct evaluations and the number of NM evaluations.

Number of iterations of Direct	Number total of direct	Before the Gathering		After the Gathering		After the Gathering		After the Gathering	
		Global Search	Local Search	Global Search	Local Search	Global Search	Local Search	Global Search	Local Search
1	21	224	224	224	224	224	224	224	224
2	39	134	134	134	134	134	134	134	134
3	73	242	242	242	242	242	242	242	242
4	103	344	344	344	344	344	344	344	344
5	145	<del>520</del>	520	<del>520</del>	520	<del>520</del>	520	<del>520</del>	520
6	197	509	793	447	731	509	793	509	793
7	257	904	1431	653	1180	904	1431	904	1431
8	323	1098	<del>1098</del>	984	<del>984</del>	1098	<del>1098</del>	1098	<del>1098</del>
9	353	1396	1572	1145	1321	1396	1572	1396	1572
10	407	1534	1786	1420	1672	1534	1786	1534	1786
11	461	1731	2172	1405	1846	1731	2172	1731	2172
12	515	1912	2444	1526	2058	1912	2444	1912	2444
13	571	1968	2637	1582	2251	1968	2637	1968	2637
14	627	2112	2781	1779	2448	2112	2781	2112	2781
15	677	2162	2831	1862	2531	2162	2831	2162	2831
16	727	2212	2995	1996	1779	2212	2995	2212	2995
17	783	2268	3051	1968	2751	2268	3051	2268	3051
18	839	2324	3107	1894	1677	2324	3107	2324	3107
19	895	2380	3320	1950	2890	2380	3320	2380	3320
20	951	2436	3373	2006	2943	2436	3373	2436	3373
21	1007	2492	3628	2029	3165	2492	3628	2492	3628
22	1031	2516	3652	2106	3242	2516	3652	2516	3652
23	1093	<del>2516</del>	<del>3652</del>	<del>2106</del>	<del>3242</del>	<del>2516</del>	<del>3652</del>	<del>2516</del>	<del>3652</del>
24	1131	<del>2516</del>	<del>3652</del>	<del>2106</del>	<del>3242</del>	<del>2516</del>	<del>3652</del>	<del>2516</del>	<del>3652</del>
25	1183	<del>2516</del>	<del>3652</del>	<del>2106</del>	<del>3242</del>	<del>2516</del>	<del>3652</del>	<del>2516</del>	<del>3652</del>
26	1233	3781	3781	<del>3781</del>	<del>3781</del>	<del>3781</del>	<del>3781</del>	<del>3781</del>	<del>3781</del>
27	1287	4612	4612	<del>4612</del>	<del>4612</del>	<del>4612</del>	<del>4612</del>	<del>4612</del>	<del>4612</del>
50	2781	<del>2781</del>	21939	3411	4157	3411	4157	3411	4233
59	3355	18099	28313	3985	4731	3985	5378	3985	5441
66	3673	26664	35731	4303	5696	4303	5696	4303	5759
74	4041	31054	41447	4671	5392	4671	6039	4671	6102
87	4689	42170	77935	5335	6699	5335	7367	5335	7430

*Figure 3 : table with the real number of evaluation running by Direct and by NM*

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Some Direct iterations are not reachable, that's why some boxes are crossed. The 5 first Direct iterations give the same number of real evaluation because with the gathering process or without, the 'mix method' selects the same number of points (see figure 2). Then for the next Direct iterations, the number of real evaluation grows up as the Direct iteration. The same observation than for the number of selecting points can be done because this real number of evaluation depends on the number of selecting points.

For the 27 first Direct iteration, the local and global search with the gathering process for  $\alpha=0.5$  and  $0.8$  have the same real number of total evaluation than without the gathering process because as seen in figure 2, the gathering process selects the same points. Only the gathering process with  $\alpha=1$  is useful for the 27 first Direct iteration. For the 50<sup>th</sup>, 59<sup>th</sup>, 66<sup>th</sup>, 74<sup>th</sup> and 87<sup>th</sup> Direct iteration, the real number of total evaluation is the same with the gathering process for the global search with  $\alpha=1, 0.8$  and  $0.5$  so we cannot say for the global search which  $\alpha$  is the most suitable. On the other hand, the real number of total evaluation for the gathering process with  $\alpha=1$  for the local search is smaller than with  $\alpha < 1$ , indeed the gathering process select less points for  $\alpha=1$  than for  $\alpha < 1$ .

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### 4 Nelder Mead average

The real number of total evaluation calculated in the previous part allows the calculation of the number of evaluation for NM. To just take the evaluation number for one NM, the following calculation has to be made:

$$\text{One Nelder Mead evaluation} = \frac{\text{counter results} - \text{Direct evaluations}}{\text{number of selecting points}} \quad (1)$$

Number of iterations of Direct	Number total of direct	alpha=1		alpha=0.8		alpha=0.5			
		Before the Gathering	Local Search	Global Search	Local Search	Global Search	Local Search	Global Search	Local Search
1	21	203	203	203	203	203	203	203	203
2	39	95	95	95	95	95	95	95	95
3	73	169	169	169	169	169	169	169	169
4	103	121	121	121	121	121	121	121	121
5	145	<del>121</del>	125	<del>121</del>	125	<del>121</del>	125	<del>121</del>	125
6	197	104	119	125	134	104	119	104	119
7	257	92	107	79	103	92	107	92	107
8	323	97	<del>107</del>	110	<del>103</del>	97	<del>107</del>	97	<del>107</del>
9	353	95	102	88	97	95	102	95	102
10	407	94	99	101	105	94	99	94	99
11	461	91	95	86	92	91	95	91	95
12	515	87	92	84	91	87	92	87	92
13	571	87	94	84	93	87	94	87	94
14	627	87	94	89	96	87	94	87	94
15	677	87	94	91	98	87	94	87	94
16	727	87	95	98	53	87	95	87	95
17	783	87	95	91	98	87	95	87	95
18	839	87	95	81	42	87	95	87	95
19	895	87	97	81	95	87	97	87	97
20	951	87	97	81	95	87	97	87	97
21	1007	87	101	79	98	87	101	87	101
22	1031	87	101	83	101	87	101	87	101
23	1093	<del>87</del>	<del>101</del>	<del>83</del>	<del>101</del>	<del>87</del>	<del>101</del>	<del>87</del>	<del>101</del>
24	1131	<del>87</del>	<del>101</del>	<del>83</del>	<del>101</del>	<del>87</del>	<del>101</del>	<del>87</del>	<del>101</del>
25	1183	<del>87</del>	<del>101</del>	<del>83</del>	<del>101</del>	<del>87</del>	<del>101</del>	<del>87</del>	<del>101</del>
26	1233	98	98	<del>83</del>	<del>101</del>	<del>87</del>	<del>101</del>	<del>87</del>	<del>101</del>
27	1287	98	98	<del>83</del>	<del>101</del>	<del>87</del>	<del>101</del>	<del>87</del>	<del>101</del>
50	2781	<del>98</del>	518	630	459	630	459	630	363
59	3355	641	543	630	459	630	506	630	417
66	3673	639	543	630	506	630	506	630	417
74	4041	643	558	630	450	630	500	630	412
87	4689	646	610	646	503	646	536	646	457
average 1		101	108	101	105	101	109	101	109
average 2		642	554	633	475	633	501	633	413

Figure 4 : table with the evaluation NM average

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In figure 4, we can see two averages, the average 1 which is the average of the real number of evaluation for one NM for the 27 first Direct iteration and the average 2 which is the average of the real number of evaluation for one NM for the 50<sup>th</sup>, 59<sup>th</sup>, 66<sup>th</sup>, 74<sup>th</sup> and 87<sup>th</sup> Direct iteration. As in the figure 3, some boxes are crossed because some Direct iterations aren't reachable.

The average 1 is almost 100 whereas the average 2 is almost 560. We did not expect this value for the average 2. Indeed when Direct has made a lot of iterations it means it has run during a long time and a lot of points have been evaluated, so the 'mix method' has a large choice to select the points. With this large choice, the selecting process selects only very good points. And normally when NM starts in good area (near good point) it should be a very efficient method but here we see that NM needs almost 560 evaluations which is not efficient and take a lot of time. This high average 2 show us a new NM problem which has never been seen before (see Annex 4).

The average 1 show us that NM just need almost 100 evaluations to get its results, this value is acceptable for our simulation based optimization problem unlikely the average 2. When Direct has just made few iteration (less than 27), the 'mix method' have less choice than if Direct had made 50 iteration. So with less information, the selecting process selects some points and NM works very fast.

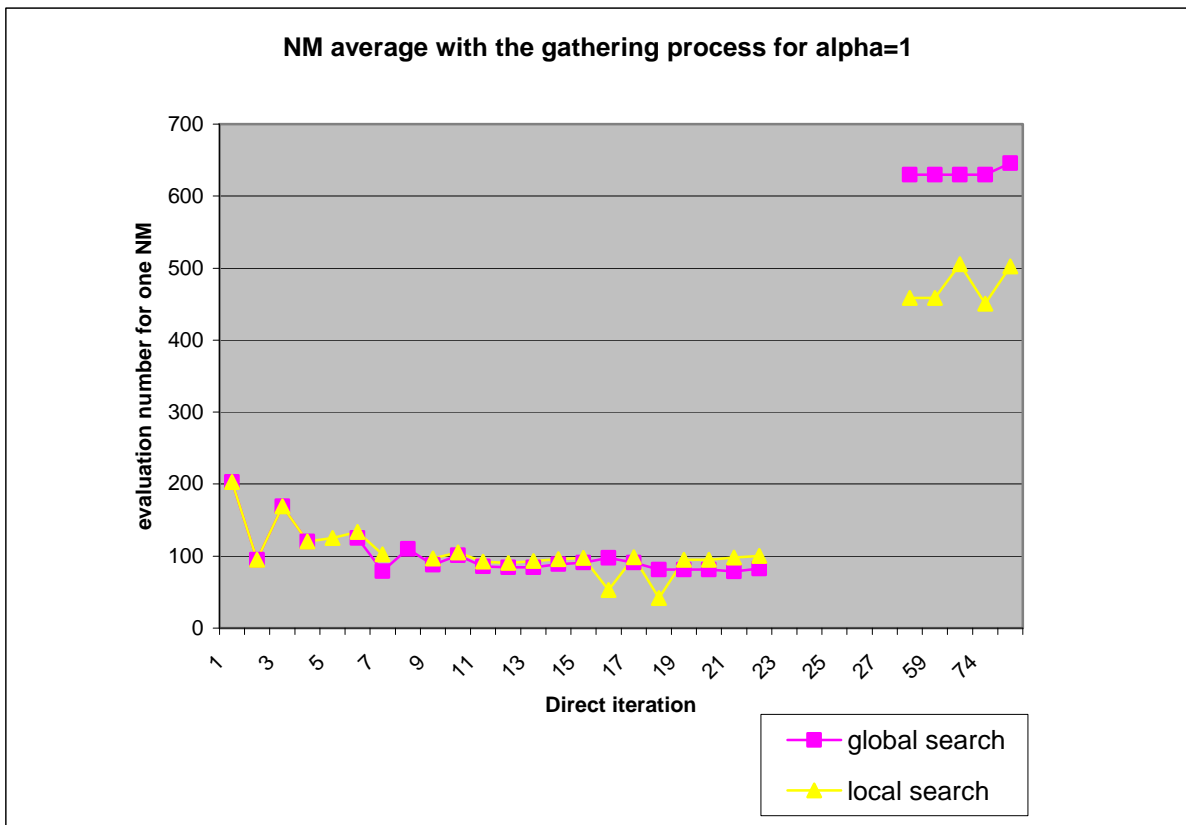


Figure 5: NM average

## Appendix C

In figure 5, we can see the NM average with the gathering process for  $\alpha=1$ . I choose this case but all the other cases without or with the gathering process show the same thing. We see that for the 23 first Direct iterations, the average is almost 100 except for the 5 first Direct iteration whereas for the 50<sup>th</sup> Direct iteration and more the average is almost 630 for the global search and 450 for the local search. The global search needs more evaluation than the local search only for high Direct iteration.

Change the average set in the program from 60 to 100 will reflect more the reality (at least for the 27 first Direct iterations). If we change this average, the number of total evaluations predicted will also change (see Annex 1 and Annex 2). This number is the one we need to launch the mix method (see figure 21: Scheme of the Mix method in ref). This number is getting by the calculation:

$$\text{Number total predicted} = \text{direct evaluation} + (\text{number of selecting point} * \text{average NM}) \quad (2)$$

The thing remarkable in looking the table in Annex 1 and Annex 2 is that with an average at 100, there are more points which are unreachable than with 60, so it will reflect more the real number of evaluation but some Direct iteration become unreachable.

## 5 The objective function value results

In figure 6, we can see the table with the objective function value results for the local and global search without and with the gathering process for the three different alpha values 1, 0.8 and 0.5. Alpha and the gathering process have no effect on the objective function results for the 5 first Direct iterations. After the 5 first Direct iterations, the gathering process has a real effect on the number of selecting points until the 22<sup>nd</sup> Direct iteration but only for  $\alpha=1$ , indeed for  $\alpha=0.8$  and 0.5 the results are the same than without the gathering process. This is visible on the figure 6, where the two columns before the gathering process are identical to the two columns after the gathering process for  $\alpha=0.8$  and 0.5 from the 1<sup>st</sup> Direct iteration to the 22<sup>nd</sup> Direct iteration. So if we want to use only the 27 first Direct iterations, we should not use the gathering process with  $\alpha < 1$  because otherwise the gathering process will not reduce the number of selecting points and so it will be not efficient. The gathering process with  $\alpha=1$  for the local and the global search has a real advantage compared to without the gathering process, indeed we can get the same results than without the gathering process (see Annex 3) with less points so with less consuming simulation time.

## Appendix C

Number of iterations of Direct	Number total of direct			alpha=1		alpha=0.8		alpha=0.5	
		Before the Gathering		After the Gathering Process		After the Gathering Process		After the Gathering Process	
		Global Search	Local Search	Global Search	Local Search	Global Search	Local Search	Global Search	Local Search
1	21	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478
2	39	12.987013	12.987013	12.987013	12.987013	12.987013	12.987013	12.987013	12.987013
3	73	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437
4	103	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437
5	145	<del>13.100437</del>	13.100437	<del>13.100437</del>	13.100437	<del>13.100437</del>	13.100437	<del>13.100437</del>	13.100437
6	197	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478
7	257	13.043478	13.043478	12.987013	12.987013	13.043478	13.043478	13.043478	13.043478
8	323	13.043478	<del>13.043478</del>	13.043478	<del>13.043478</del>	13.043478	<del>13.043478</del>	13.043478	<del>13.043478</del>
9	353	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478
10	407	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478
11	461	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478	13.043478
12	515	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
13	571	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
14	627	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
15	677	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
16	727	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
17	783	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
18	839	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
19	895	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
20	951	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
21	1007	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
22	1031	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437	13.043478	13.100437
23	1093	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>
24	1131	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>
25	1183	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>
26	1233	13.043478	13.043478	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>
27	1287	13.043478	13.043478	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>	<del>13.043478</del>
50	2781	<del>13.100437</del>	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437
59	3355	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437
66	3673	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437
74	4041	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437	13.100437
87	4689	13.100437	13.100437	13.100437	13.043478	13.100437	13.100437	13.100437	13.100437

Figure 6 : table with the objective function results

## Appendix C

In looking for the value found by the 'mix method' for the 22 first Direct iteration, we see that the global search can find the best value 13.100437 at the 3<sup>rd</sup> and 4<sup>th</sup> Direct iteration but then the best value found is only 13.043478 until the 22<sup>nd</sup> Direct iteration whereas the local search find a lot of time 13.100437 during the 22 first Direct iteration. Here we see that the local search is more efficient than the global search to find the best objective function value for the 22 first Direct iterations. This is visible on the figure 7 below for the gathering process with  $\alpha=1$  but the same thing is remarkable with other  $\alpha$  value and without the gathering process.

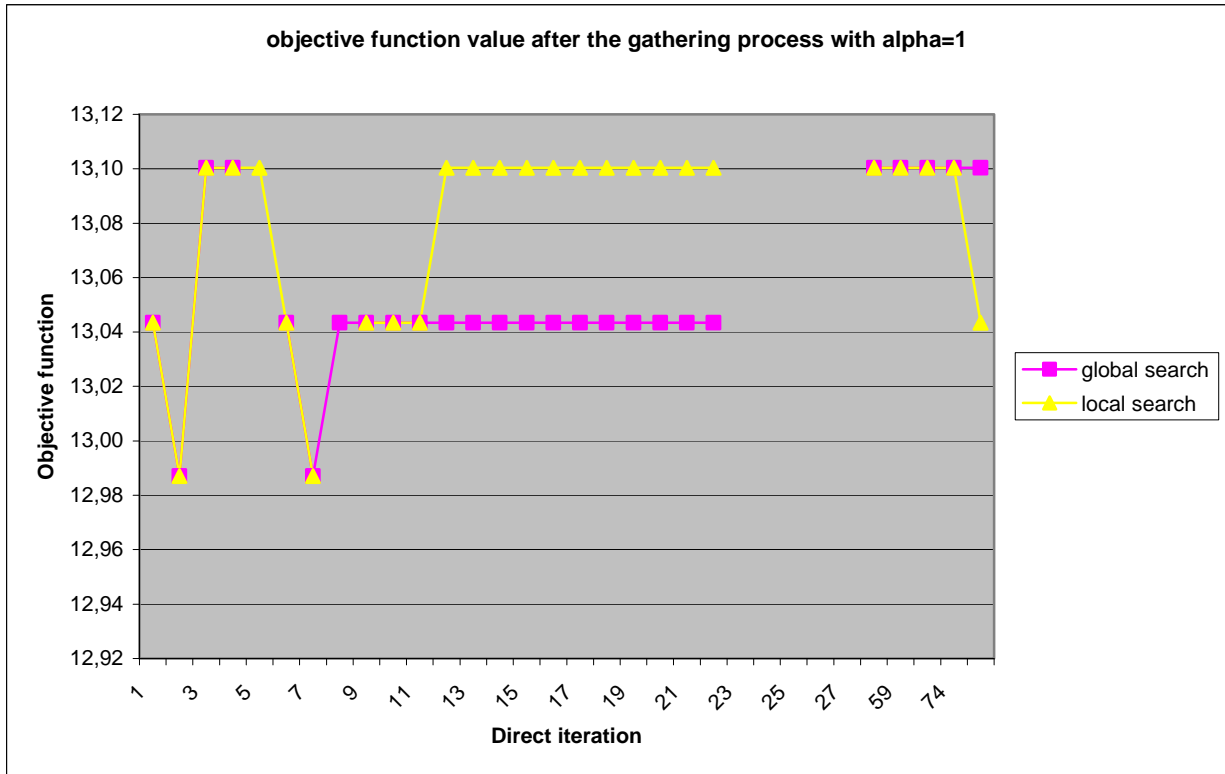


Figure 7: objective function value after the gathering process with  $\alpha=1$

After the 50<sup>th</sup> Direct iteration, the local search and the global search find the same best objective function value with and without the gathering process (see Annex 3), we can say that the gathering process is efficient for the three  $\alpha$  values but  $\alpha=1$  is the most efficient because the gathering process with  $\alpha=1$  selects less points than with  $\alpha=0.8$  and  $0.5$ . Unlikely at the 27 first Direct iteration, the global search always find the same best value 13.100437 than the local search and as the selecting process for the global search select less points than for the local search, the global search is more efficient than the local search after the 50<sup>th</sup> Direct iteration to find the best objective function value. There is a strange result for the iteration 87 after the gathering process for  $\alpha=1$  in the local search, indeed the global search finds a better result than the local search which is normally impossible. If we look the number of selecting points for the 87<sup>th</sup> Direct iteration, we that in global search it gathers all the selecting points only in one point whereas in local search in four points. The NM for the point found by the global search finds a better result than the NM for the 4 points. Here we see the same NM problem said in [ref 1] (chapter 5.6, different slopes of the Objective function in one simplex).

## Appendix C

The global search with the gathering process for high Direct iteration shows that all the points having the maximum value 13.100437 are in a small area because the gathering process gathers them in one point. So it means that Direct has not yet found the value 13.100437 elsewhere in the search space. But Direct can maybe reaches it with more iterations and then the NM for the new good points found by Direct could maybe provide very interesting results.

As the final goal is to find the function optimum, the gathering process with  $\alpha=1$  and with the local search should be used if we only want to launch the 22 first Direct iterations. Otherwise, to launch the 'mix method' with Direct iteration superior to 50, we should use the gathering process with  $\alpha=1$  and with the global search. There is one case where the gathering process doesn't find the same objective function value than without the gathering process, it 's for the 7<sup>th</sup> Direct iteration. So if we want to launch the 7<sup>th</sup> Direct iteration we should use the selecting process without the gathering process.

An other goal was to see if it 's better to launch few Direct iteration and then launch NM for the selecting points or if it is preferable to launch a lot of Direct iteration and then launch the NM for the selecting points. The best point value has been found at the 3<sup>rd</sup> Direct iteration with 242 total evaluations so for this objective function, we don't need to launch Direct for a lot of iterations to find the best point. We have seen that for small Direct iterations, there are a lot of selecting points with the gathering process, more than for high Direct iterations (figure 2). But we have also seen that the NM average for high Direct iteration is almost 560 whereas it is only 100 for small Direct iteration. So even if there is just few points selected for high Direct iteration, as their NM evaluations is more than 5 times bigger than for small Direct iteration, the 'mix method' is not very time efficient for high Direct iteration and should be used with small Direct iteration.



### Conclusion

The VM at the Production Technology Centre in TROLLHATTAN has been used to test the 'mix method' in order to see if this method could be efficient for a simulation based optimization problem.

The selecting process is the key part of the 'mix method' and can be achieved in using the gathering process or not. The goal of the gathering process is to gather the interesting selecting points in package called chains. The gathering process has a good behaviour with  $\alpha=1$ , 0.8 and 0.5. Indeed, it reduces considerably the number of selecting points. The most efficient  $\alpha$  value for this simulation based optimization problem is  $\alpha=1$  because it selects less point than with  $\alpha=0.8$  and 0.5 and because the results with  $\alpha=1$  are the same than if all the selecting points were starting points for NM. So for the objective function actually used, the gathering process with  $\alpha=0.8$  and 0.5 are unsuitable. The gathering process is a real advantage for the 'mix method' because it allows winning time and in the same time getting the best point.

For the 22 first Direct iteration, the gathering process with  $\alpha=1$  and with the local search reach the expected behaviour except for 7<sup>th</sup> Direct iteration where it 's better to not use the gathering process. For high Direct iteration, the gathering process with the global search should be used because the global search selects less point than the local search.

The 'mix method' has been launched until the 87<sup>th</sup> Direct iteration and has not found a new optimum but it 's maybe possible in launching more Direct iteration.

### Bibliography:

Ref 1 . Simulation based optimization using a combination of two optimization methods, Damien RINGENBACH,2009

## Appendix C

### Annex 1:

This annex 1 is the extension of the figure 24 in [ref 1] with more Direct iteration and with different alpha, it presents the number of total evaluations predicted with the NM average 60. This number is necessary to launch the 'mix method' if the average in the program is 60.

alpha=1

alpha=0.8

alpha=0.5

Number of iterations of Direct	Number total of direct	Before the Gathering		After the Gathering		After the Gathering		After the Gathering	
		Global Search	Local Search	Global Search	Local Search	Global Search	Local Search	Global Search	Local Search
1	21	81	81	81	81	81	81	81	81
2	39	99	99	99	99	99	99	99	99
3	73	133	133	133	133	133	133	133	133
4	103	223	223	223	223	223	223	223	223
5	145	<del>325</del>	325	<del>325</del>	325	<del>325</del>	325	<del>325</del>	325
6	197	377	497	317	437	377	497	377	497
7	257	677	917	557	797	677	917	677	917
8	323	803	<del>1073</del>	683	<del>953</del>	803	<del>1073</del>	803	<del>1073</del>
9	353	1013	1073	893	953	1013	1073	1013	1073
10	407	1127	1247	1007	1127	1127	1247	1127	1247
11	461	1301	1541	1121	1361	1301	1541	1301	1541
12	515	1475	1775	1235	1535	1475	1775	1475	1775
13	571	1531	1891	1291	1651	1531	1891	1531	1891
14	627	1647	2007	1407	1767	1647	2007	1647	2007
15	677	1697	2057	1457	1817	1697	2057	1697	2057
16	727	1747	2167	1507	1927	1747	2167	1747	2167
17	783	1803	2223	1563	1983	1803	2223	1803	2223
18	839	1859	2279	1619	2039	1859	2279	1859	2279
19	895	1915	2395	1675	2155	1915	2395	1915	2395
20	951	1971	2451	1731	2211	1971	2451	1971	2451
21	1007	2027	2567	1787	2327	2027	2567	2027	2567
22	1031	2051	2591	1811	2351	2051	2591	2051	2591
23	1093	<del>2793</del>	<del>2793</del>	<del>2793</del>	<del>2793</del>	<del>2793</del>	<del>2793</del>	<del>2793</del>	<del>2793</del>
24	1131	<del>3327</del>	<del>3327</del>	<del>3327</del>	<del>3327</del>	<del>3327</del>	<del>3327</del>	<del>3327</del>	<del>3327</del>
25	1183	<del>3919</del>	<del>3919</del>	<del>3919</del>	<del>3919</del>	<del>3919</del>	<del>3919</del>	<del>3919</del>	<del>3919</del>
26	1233	2793	2793	<del>2793</del>	<del>2793</del>	<del>2793</del>	<del>2793</del>	<del>2793</del>	<del>2793</del>
27	1287	3327	3327	<del>3327</del>	<del>3327</del>	<del>3327</del>	<del>3327</del>	<del>3327</del>	<del>3327</del>
28	1339	3919	3919	<del>3919</del>	<del>3919</del>	<del>3919</del>	<del>3919</del>	2419	<del>3919</del>
29	1391	<del>4141</del>	<del>4141</del>	<del>4141</del>	<del>4141</del>	<del>4141</del>	<del>4141</del>	<del>4141</del>	<del>4141</del>
30	1441	<del>4187</del>	4141	<del>4187</del>	<del>4187</del>	<del>4187</del>	<del>4187</del>	<del>4187</del>	2821
31	1487	<del>4855</del>	4187	<del>4855</del>	<del>4855</del>	<del>4855</del>	<del>4855</del>	<del>4855</del>	2867
32	1559	<del>2483</del>	<del>2483</del>	<del>2483</del>	<del>2483</del>	<del>2483</del>	<del>2483</del>	<del>2483</del>	<del>2483</del>
33	1633	<del>2563</del>	<del>2563</del>	<del>2563</del>	<del>2563</del>	<del>2563</del>	<del>2563</del>	<del>2563</del>	<del>2563</del>
34	1073	<del>2683</del>	<del>2683</del>	<del>2683</del>	<del>2683</del>	<del>2683</del>	<del>2683</del>	<del>2683</del>	<del>2683</del>
35	1767	<del>2629</del>	<del>2629</del>	<del>2629</del>	<del>2629</del>	<del>2629</del>	<del>2629</del>	<del>2629</del>	<del>2629</del>
36	1829	<del>2749</del>	<del>2749</del>	<del>2749</del>	<del>2749</del>	<del>2749</del>	<del>2749</del>	<del>2749</del>	<del>2749</del>
37	1893	<del>2695</del>	<del>2695</del>	<del>2695</del>	<del>2695</del>	<del>2695</del>	<del>2695</del>	<del>2695</del>	<del>2695</del>
38	1957	<del>2815</del>	<del>2815</del>	<del>2815</del>	<del>2815</del>	<del>2815</del>	<del>2815</del>	<del>2815</del>	<del>2815</del>
39	2017	<del>2875</del>	<del>2875</del>	<del>2875</del>	<del>2875</del>	<del>2875</del>	<del>2875</del>	<del>2875</del>	<del>2875</del>
40	2069	<del>2815</del>	<del>2815</del>	<del>2815</del>	<del>2815</del>	2077	<del>2815</del>	<del>2815</del>	<del>2815</del>
41	2141	<del>2483</del>	<del>2483</del>	<del>2483</del>	<del>2483</del>	2129	<del>2483</del>	<del>2483</del>	<del>2483</del>
42	2197	<del>2563</del>	<del>2563</del>	<del>2563</del>	<del>2563</del>	2201	<del>2563</del>	<del>2563</del>	<del>2563</del>
43	2269	<del>2201</del>	<del>2201</del>	<del>2201</del>	<del>2201</del>	2257	<del>2201</del>	<del>2201</del>	<del>2201</del>
44	2351	<del>2257</del>	<del>2257</del>	<del>2257</del>	<del>2257</del>	2329	<del>2257</del>	<del>2257</del>	<del>2257</del>
45	2423	<del>2329</del>	<del>2329</del>	<del>2329</del>	<del>2329</del>	2411	<del>2329</del>	<del>2329</del>	<del>2329</del>
46	2503	<del>2411</del>	<del>2411</del>	<del>2411</del>	<del>2411</del>	2411	<del>2411</del>	<del>2411</del>	<del>2411</del>
47	2569	<del>2483</del>	4283	<del>2483</del>	2483	2483	<del>2483</del>	2483	<del>2483</del>
48	2635	<del>2563</del>	4543	<del>2563</del>	2563	2563	<del>2563</del>	2563	<del>2563</del>
		<del>2683</del>	4729	<del>2683</del>	2683	2683	<del>2683</del>	2683	<del>2683</del>
		<del>2629</del>	4855	<del>2629</del>	2629	2629	<del>2629</del>	2629	<del>2629</del>
		<del>2749</del>		<del>2749</del>	2749	2749	<del>2749</del>	2749	<del>2749</del>
		<del>2695</del>		<del>2695</del>	2695	2695	<del>2695</del>	2695	<del>2695</del>
		<del>2815</del>		<del>2815</del>	2815	2815	<del>2815</del>	2815	<del>2815</del>
		<del>2875</del>		<del>2875</del>	2875	2875	<del>2875</del>	2875	<del>2875</del>

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49	2713		4933	2773	2893	2773	2893	2773	2953
50	2781		5001	2841	2961	2841	2961	2841	3021
51	2847	3987	5247	2907	3027	2907	3027	2907	3087
52	2913	4113	5373	2973	3093	2973	3093	2973	3153
53	2979	4179	5439	3039	3159	3039	3159	3039	3219
54	3045	4245	5505	3105	3225	3105	3225	3105	3285
55	3109	4369	5689	3169	3289	3169	3289	3169	3349
56	3173	4433	5753	3233	3353	3233	3353	3233	3413
57	3237	4497	5817	3297	3417	3297	3417	3297	3477
58	3303	4623	5943	3363	3483	3363	3483	3363	3543
59	3355	4735	6115	3415	3535	3415	3595	3415	3655
60	3401	4841	6221	3461	3581	3461	3641	3461	3701
61	3445	4885	6265	3505	3625	3505	3685	3505	3745
62	3489	4929	6309	3549	3669	3549	3729	3549	3789
63	3535	5155	6535	3595	3775	3595	3775	3595	3835
64	3581	5381	6761	3641	3821	3641	3821	3641	3881
65	3627	5607	6987	3687	3867	3687	3867	3687	3927
66	3673	5833	7213	3733	3913	3733	3913	3733	3973
67	3719	5939	7379	3779	3899	3779	3959	3779	4019
68	3765	6045	7485	3825	3945	3825	4005	3825	4065
69	3811	6151	7651	3871	3991	3871	4051	3871	4111
70	3865	6265	7765	3925	4045	3925	4105	3925	4165
71	3909	6309	7809	3969	4089	3969	4149	3969	4209
72	3953	6413	7913	4013	4133	4013	4193	4013	4253
73	3997	6457	7957	4057	4177	4057	4237	4057	4297
74	4041	6561	8061	4101	4221	4101	4281	4101	4341
75	4095	6615	8115	4155	4275	4155	4335	4155	4395
76	4139	6719	8219	4199	4319	4199	4379	4199	4439
77	4197	6777	8277	4257	4377	4257	4437	4257	4497
78	4241	6881	8381	4301	4421	4301	4481	4301	4541
79	4281	6921	8421	4341	4461	4341	4521	4341	4581
80	4321	6961	8461	4381	4501	4381	4561	4381	4621
81	4389	7029	8529	4449	4569	4449	4629	4449	4689
82	4445	7085	8585	4505	4625	4505	4685	4505	4745
83	4493	7133	8633	4553	4673	4553	4733	4553	4793
84	4539	7179	8679	4599	4719	4599	4779	4599	4839
85	4585	7225	8725	4645	4765	4645	4825	4645	4885
86	4629	7269	8769	4689	4809	4689	4869	4689	4929
87	4689	8169	11889	4749	4929	4749	4989	4749	5049
88	4747	9127	13387	4807	4987	4807	5047	4807	5167
89	4815	10035	14295	4875	5055	4875	5115	4875	5235
90	4875	10995	15255	4935	5115	4935	5175	4935	5295
91	4935	11955	16695	4995	5175	4995	5235	4995	5355
92	5017	12997	17737	5077	5257	5077	5317	5077	5437
93	5077	13897	18637	5137	5317	5137	5377	5137	5497
94	5135	14915	19655	5195	5375	5195	5435	5195	5555
95	5193	15873	20613	5253	5433	5253	5493	5253	5613
96	5261	16901	21641	5321	5501	5321	5561	5321	5681
97	5329	17809	22609	5389	5629	5389	5689	5389	5809
98	5395	18835	23635	5455	5695	5455	5755	5455	5875
99	5463	19803	24663	5523	5823	5523	5883	5523	6003
100	5529	20829	25689	5589	5889	5589	5949	5589	6069

Figure 8 : table with the number of total evaluations predicted with the NM average 60.

## Appendix C

### Annex 2:

This annex 2 presents the number of total evaluations predicted as the Annex 1 but with the NM average 100. This average will reflect more the real number of evaluation for NM.

alpha=1

alpha=0.8

alpha=0.5

Number of iterations of Direct	Number total of direct	Before the Gathering		After the Gathering		After the Gathering		After the Gathering	
		Global Search	Local Search	Global Search	Local Search	Global Search	Local Search	Global Search	Local Search
1	21	121	121	121	121	121	121	121	121
2	39	139	139	139	139	139	139	139	139
3	73	173	173	173	173	173	173	173	173
4	103	303	303	303	303	303	303	303	303
5	145	<del>303</del>	445	<del>303</del>	445	<del>303</del>	445	<del>303</del>	445
6	197	497	697	397	597	497	697	497	697
7	257	957	1357	757	1157	957	1357	957	1357
8	323	1123	<del>1357</del>	923	<del>1157</del>	1123	<del>1357</del>	1123	<del>1357</del>
9	353	1453	1553	1253	1353	1453	1553	1453	1553
10	407	1607	1807	1407	1607	1607	1807	1607	1807
11	461	1861	2261	1561	1961	1861	2261	1861	2261
12	515	2115	2615	1715	2215	2115	2615	2115	2615
13	571	2171	2771	1771	2371	2171	2771	2171	2771
14	627	2327	2927	1927	2527	2327	2927	2327	2927
15	677	2377	2977	1977	2577	2377	2977	2377	2977
16	727	2427	3127	2027	2727	2427	3127	2427	3127
17	783	2483	3183	2083	2783	2483	3183	2483	3183
18	839	2539	3239	2139	2839	2539	3239	2539	3239
19	895	2595	3395	2195	2995	2595	3395	2595	3395
20	951	2651	3451	2251	3051	2651	3451	2651	3451
21	1007	2707	3607	2307	3207	2707	3607	2707	3607
22	1031	2731	3631	2331	3231	2731	3631	2731	3631
23	1093	<del>2731</del>	<del>3631</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>2731</del>	<del>3631</del>
24	1131	<del>2731</del>	<del>3631</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>2731</del>	<del>3631</del>
25	1183	<del>2731</del>	<del>3631</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>2731</del>	<del>3631</del>
26	1233	3833	3833	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>2731</del>	<del>3631</del>
27	1287	4687	4687	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>2731</del>	<del>3631</del>
28	1339	5639	5639	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	3139	<del>3631</del>
29	1391	<del>5639</del>	<del>5639</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3631</del>
30	1441	<del>5639</del>	5941	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	3741
31	1487	<del>5639</del>	5987	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	3787
32	1559	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
33	1633	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
34	1073	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
35	1767	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
36	1829	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
37	1893	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
38	1957	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
39	2017	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
40	2069	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
41	2141	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
42	2197	<del>5639</del>	<del>5987</del>	<del>2331</del>	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
43	2269	<del>5639</del>	<del>5987</del>	2369	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
44	2351	<del>5639</del>	<del>5987</del>	2451	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
45	2423	<del>5639</del>	<del>5987</del>	2523	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
46	2503	<del>5639</del>	<del>5987</del>	2603	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
47	2569	<del>5639</del>	6169	2669	<del>3231</del>	<del>2731</del>	<del>3631</del>	<del>3139</del>	<del>3787</del>
48	2635	<del>5639</del>	6335	2735	<del>3231</del>	2735	<del>3631</del>	<del>3139</del>	<del>3787</del>

## Appendix C

49	2713		6413	2813		2813			
50	2781		6481	2881		2881			
51	2847		6847	2947		2947			
52	2913		7013	3013		3013			
53	2979		7079	3079	3279	3079			
54	3045		7145	3145	3345	3145		3145	
55	3109		7409	3209	3409	3209		3209	
56	3173		7473	3273	3473	3273		3273	
57	3237		7537	3337	3537	3337		3337	
58	3303		7703	3403	3603	3403		3403	
59	3355	5655	7955	3455	3655	3455	3755	3455	3855
60	3401	5801	8101	3501	3701	3501	3801	3501	3901
61	3445	5845	8145	3545	3745	3545	3845	3545	3945
62	3489	5889	8189	3589	3789	3589	3889	3589	3989
63	3535	6235	8535	3635	3935	3635	3935	3635	4035
64	3581	6581	8881	3681	3981	3681	3981	3681	4081
65	3627	6927	9227	3727	4027	3727	4027	3727	4127
66	3673	7273	9573	3773	4073	3773	4073	3773	4173
67	3719	7419	9819	3819	4019	3819	4119	3819	4219
68	3765	7565	9965	3865	4065	3865	4165	3865	4265
69	3811	7711	10211	3911	4111	3911	4211	3911	4311
70	3865	7865	10365	3965	4165	3965	4265	3965	4365
71	3909	7909	10409	4009	4209	4009	4309	4009	4409
72	3953	8053	10553	4053	4253	4053	4353	4053	4453
73	3997	8097	10597	4097	4297	4097	4397	4097	4497
74	4041	8241	10741	4141	4341	4141	4441	4141	4541
75	4095	8295	10795	4195	4395	4195	4495	4195	4595
76	4139	8439	10939	4239	4439	4239	4539	4239	4639
77	4197	8497	10997	4297	4497	4297	4597	4297	4697
78	4241	8641	11141	4341	4541	4341	4641	4341	4741
79	4281	8681	11181	4381	4581	4381	4681	4381	4781
80	4321	8721	11221	4421	4621	4421	4721	4421	4821
81	4389	8789	11289	4489	4689	4489	4789	4489	4889
82	4445	8845	11345	4545	4745	4545	4845	4545	4945
83	4493	8893	11393	4593	4793	4593	4893	4593	4993
84	4539	8939	11439	4639	4839	4639	4939	4639	5039
85	4585	8985	11485	4685	4885	4685	4985	4685	5085
86	4629	9029	11529	4729	4929	4729	5029	4729	5129
87	4689	10489	16689	4789	5089	4789	5189	4789	5289
88	4747	12047	19147	4847	5147	4847	5247	4847	5447
89	4815	13515	20615	4915	5215	4915	5315	4915	5515
90	4875	15075	22175	4975	5275	4975	5375	4975	5575
91	4935	16635	24535	5035	5335	5035	5435	5035	5635
92	5017	18317	26217	5117	5417	5117	5517	5117	5717
93	5077	19777	27677	5177	5477	5177	5577	5177	5777
94	5135	21435	29335	5235	5535	5235	5635	5235	5835
95	5193	22993	30893	5293	5593	5293	5693	5293	5893
96	5261	24661	32561	5361	5661	5361	5761	5361	5961
97	5329	26129	34129	5429	5829	5429	5929	5429	6129
98	5395	27795	35795	5495	5895	5495	5995	5495	6195
99	5463	29363	37463	5563	6063	5563	6163	5563	6363
100	5529	31029	39129	5629	6129	5629	6229	5629	6429

Figure 9 : table with the number of total evaluations predicted with the NM average 100.

**Annex 3:**

The Annex 3 presents the objective function value for the different selecting processes with the local search and the global search as in figure 6. It shows that there is almost no difference between the different selecting processes for the objective function value if we use the local search (figure 10) and there is also no difference if we use the global search (figure 11). Indeed, we can see that most of the points are superimposed which means that the ‘mix method’ finds the same result if it uses the gathering process with different alpha values than without. Differences appear if it uses the global search or the local search (figure 7).

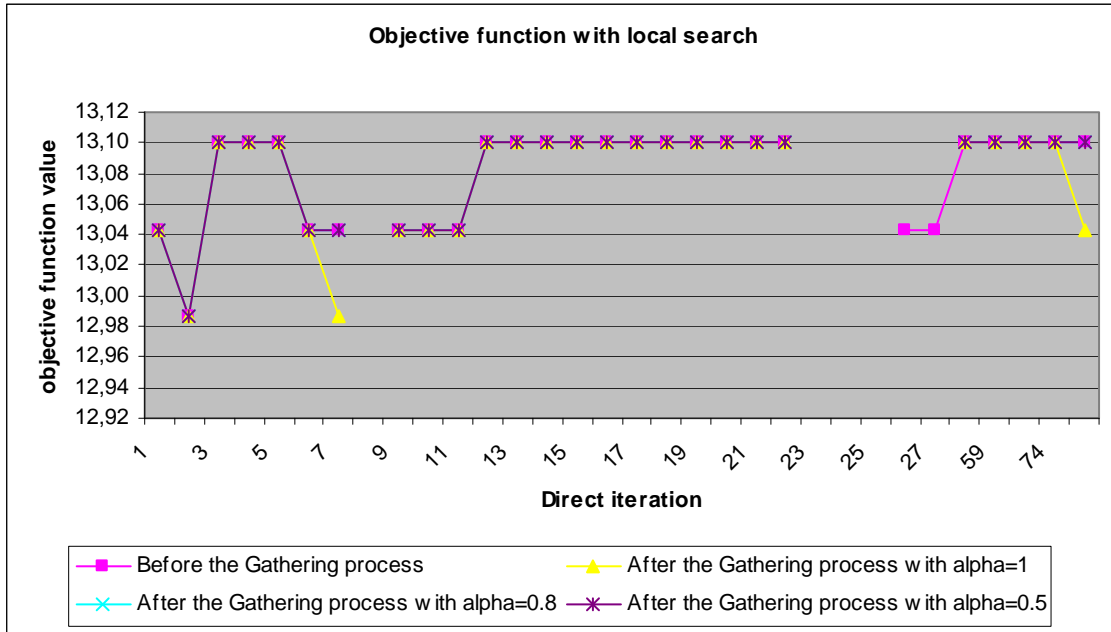


Figure 10: objective function with local search for the different selection processes

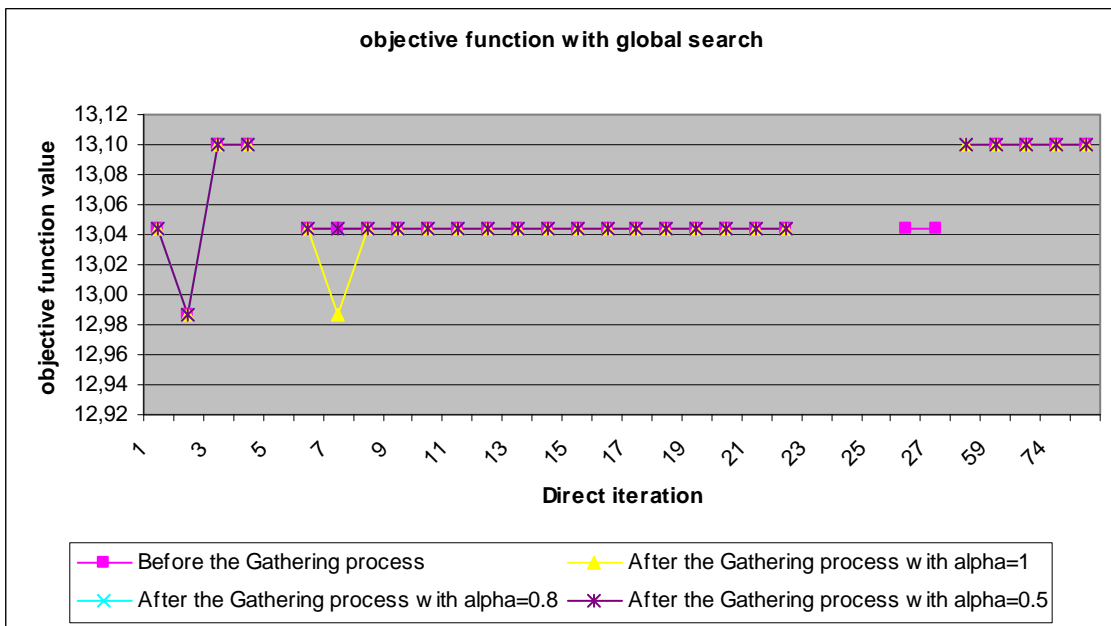


Figure 11: objective function with global search for the different selection processes

## Annex 4:

The Annex 4 presents a NM problem. As said in chapter 2 [ref 1], NM is a method which create a simplex. In two dimensions, the simplex is a triangle like in figure 12.

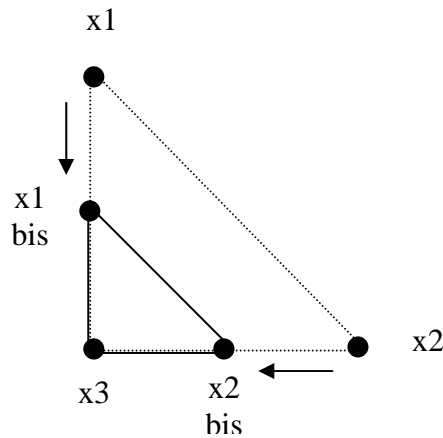


Figure 12: shrink step

For example:

- $x_3=(0,0)$  and  $fobj(x_3)=13.1$
- $x_1=(0,1)$  and  $fobj(x_1)=12.8$
- $x_2=(1,0)$  and  $fobj(x_2)=12.8$

In this case, NM should do what we call a shrink step that is to say it will move the points  $x_2$  and  $x_1$  to come close to  $x_3$  in dividing the distance between  $x_1$  and  $x_3$  by 2 and also the distance between  $x_2$  and  $x_3$ . After the NM step we normally have:

- $x_3=(0,0)$  and  $fobj(x_3)=13.1$
- $x_1 \text{ bis}=(0, \frac{1}{2})$  and  $fobj(x_1 \text{ bis})$
- $x_2 \text{ bis}=(\frac{1}{2},0)$  and  $fobj(x_2 \text{ bis})$

This is how it has to work, but in our simulation based optimization problem as the simulation can only use integers, the coordinate of  $x_1$  and  $x_2$  cannot be  $\frac{1}{2}$  so  $x_1$  and  $x_2$  don't move and NM can repeat this operation a lot of time before trying an other solution. This is an implementation mistake of the NM algorithm cause of the use of integers in the simulation. The problem in our case is in 10 dimensions so the shrink step tries to move 10 points but does not succeed. This raises the number of evaluation of 10 for each shrink step and that is why the average 2 in figure 4 is so high.